Multi-Layer Model Enabled Fault Location for Underground Cables in MMC-HVDC Grids Considering Distributed and Frequency Dependent Line Parameters

Dayou Lu, Student Member, IEEE, Yu Liu, Senior Member, IEEE, Jiahao Xie, Student Member, IEEE, Binglin Wang, Student Member, IEEE, and Qifeng Liao

Abstract — Accurate fault location for underground cables in MMC-HVDC grids reduces the time finding faults as well as the system operational costs. For MMC-HVDC grids, the data window during faults is usually short. During such severe transients, the frequency dependent line parameters of the underground cables have strong influence on the fault location accuracy. In this paper, a novel fault location algorithm for underground cables in MMC-HVDC grids is proposed. First, a multi-layer model for the underground cable using partial differential equations is proposed, which considers the distributed and frequency dependent line parameters in time domain. Afterwards, the numerical finite difference solution of the multi-layer model is proposed to obtain the voltage distributions along the entire cable. Finally, the voltage method is applied to locate faults. Numerical experiments show that the multi-layer model accurately considers the frequency dependency of the cable parameters, and the solution of the model obtains accurate voltage distribution even during system transient. Extensive test cases prove that the proposed algorithm locates faults with less errors than the existing algorithm. The method is also compatible with a relatively low sampling rate (20 kHz) and a short data window (5 ms).

Index Terms — fault location, HVDC underground cable lines, distributed and frequency dependent line parameters, short data window.

I. INTRODUCTION

WITH the development of modern power transmission systems, high voltage direct current (HVDC) transmission lines are with increasing importance due to the advantages such as no transient stability issues, flexible power flow, compatible with renewable generations, low power loss for long distance transmission, etc. Modular multilevel converter (MMC) based HVDC transmission lines are preferable due to the high controllability, high modularity, high power quality, and low switching loss [1-2]. In addition, the UnderGround Cables (UGCs) are widely adopted in HVDC system since they are more suitable for congested urban areas/submarine applications, with lower maintenance costs, fewer faults and less electromagnetic pollution compare to the OverHead transmission Lines (OHLs).

For the faults occurring inside the MMC-HVDC grids equipped with DC circuit breakers, the DC circuit breakers usually need to operate immediately after the fault (in the order of milliseconds), to ensure minimum power outage of the grid [3]. After the isolation of the fault, accurate fault location schemes can reduce the time finding the fault and improve power supply reliability. The fast operation of DC circuit breakers results in very short time window of voltage and current measurements during the fault (also in the order of milliseconds), and therefore causing challenges for fault location. Specifically, fault location algorithms are especially important for HVDC UGCs since the cables are underground and it is much more difficult for the utility crews to “visually” identify the fault compared to OHLs. Besides, the structures/materials of the UGCs are quite different from those of the OHLs, resulting in more severe frequency dependent characteristics during faults for UGCs. These facts bring additional fault location difficulties for UGCs [4].

The fault location techniques for HVDC UGCs can be mainly classified into offline fault location techniques and online fault location techniques. Offline fault location techniques determine the fault location through certain offline experiments after the fault is isolated from the system. One group of widely adopted methods is the reflectometry methods [5-6], which locate the fault by injecting a reference signal to cable and analyzing the behavior of the reflected signal. The effectiveness of these methods usually depends on careful design of injected signal and reliable wave reflection at the fault location (which could be affected by high fault resistance). Moreover, the offline fault location techniques are usually time consuming, require additional testing equipment [7], etc.

Online fault location techniques usually detect the fault location using the terminal voltages and currents during the fault. For HVDC applications, since there exists no “fundamental frequency”, phasor domain methods (widely adopted for HVAC applications [8]) may not be applicable and therefore time domain methods (using instantaneous measurements) are often adopted. These methods can be further categorized into traveling wave based methods and time domain model based methods. Traveling wave based methods [9-13] locate the fault by detecting the arrival time of the traveling waves at terminals of the line. The methods are straightforward, however, they have the following challenges that are difficult to overcome, including reliable detection of wavefronts in extreme cases (weak traveling waves due to high fault resistance, etc.) and extremely high sampling rates for accurate fault location (with typical order of MHz). In addition,
The paper proposes a multi-layer model of the line; the proposed to numerically solve the PDEs and to achieve accurate voltages. Researchers proposed a physical laws of the circuit -experiments, which are circuit. To (1) is the Bergeron model. The proposed model is formulated via the following PDEs. Afterwards, by observing the special asymmetrical structure of the multi-layer model, the solution to the multi-layer model, and the fault location results. Section VII further provides discussions on various factors. Section VIII concludes the paper.

II. REVIEW OF CLASSIC UGC MODELS

The circuit of classic UGC model with consideration of distributed and frequency independent parameters is shown in Figure 1. The circuit is a two pole circuit. The line length is l. An infinitesimal line segment at location x (from terminal k) is shown in the figure. The line segment length is dx. The two pole series resistance, series inductance, and shunt capacitance matrices per unit length are R, L and C, respectively (the shunt conductance matrix is ignored). The voltage vector and current vector are defined as \( \mathbf{u}(x,t) = [u_p(x,t) \ u_s(x,t)]^T \) and \( \mathbf{i}(x,t) = [i_p(x,t) \ i_s(x,t)]^T \), respectively.

![Figure 1. Circuit of the classic UGC model](https://example.com/circuit.png)

From Figure 1, the physical laws of the circuit is described via the following PDEs in (1),

\[
\begin{align*}
\frac{\partial u(x,t)}{\partial t} + L \frac{\partial i(x,t)}{\partial t} + R i(x,t) &= 0 \quad (1a) \\
\frac{\partial i(x,t)}{\partial t} + C \frac{\partial u(x,t)}{\partial t} &= 0 \quad (1b)
\end{align*}
\]

where parameter matrices \( R, L \) and \( C \) are frequency independent, with the dimension of 2. The dimensions of equations (1a) and (1b) are also 2.

One classic solution to (1) is the Bergeron model [18] with Karrenbauer’s transformation. It applies the Karrenbauer’s transformation to decouple the 2-dimensional PDEs into two independent PDEs for the two modes (mode \( a \) and mode 0). The Bergeron model first ignores the line resistance to obtain an analytical solution. Then the resistance distributed through the line is lumped at line terminals and the mid-point. The solution of the terminal voltage for mode \( j \) (\( j = a \) or 0) is shown in (2). The definitions of the variables are consistent with Figure 1 except that the superscript (j) corresponds to the variable of mode j.

- The paper proposes a multi-layer model in PDEs with constant coefficients; the proposed model accurately describes the physical laws that the UGC obeys, including distributed and frequency dependent parameters; the proposed model enables convenient solution procedure for UGC fault location application.
- A three-step finite difference method is proposed to systematically solve the model and to achieve accurate voltage distribution; guidance on selection of numerical intervals is provided to enable stable solutions.
- The proposed algorithm has less errors than the existing algorithm using the frequency independent UGC model; it only requires a 5 ms data window and 20 kHz sampling rate for accurate fault location.

The rest of the paper is arranged as follows. Section II briefly reviews the classic frequency independent UGC model. Section III constructs the UGC multi-layer model with the equivalent circuit diagram and the PDEs. Section IV derives the numerical solution to the multi-layer model PDEs. Section V explains the fault location algorithm. Section VI numerically verifies the accuracy of the multi-layer model, the solution to the multi-layer model, and the fault location results. Section VII further provides discussions on various factors. Section VIII concludes the paper.

compare to OHLs, the wavefronts during faults in UGCs could be much attenuated due to more severe frequency dependent characteristics of UGCs, causing further difficulties for the detection of wavefronts [3].

Time domain model based methods locate the fault via the inherent relationship between the measurements and the fault location described by a certain line model. One class of model based methods is the equation solving methods, where the fault location is first introduced as an unknown variable in the equation of the faulted line model, and afterwards determined via solving the equation. Literature [14] adopts the lumped R-L model to construct the fault location equations. Literatures [15-17] build the multi-section π model with fault via ordinary differential equations (ODEs), and afterwards the fault location is solved through dynamic state estimation. Another class of model based methods is the voltage methods. The voltage methods locate the fault by accurately calculating the voltage distribution along the line; the intersection of the voltage distributions calculated from two terminals of the line respectively shows the fault location. To solve the voltage distribution, researchers proposed methods based on the Bergeron model of the line [18-19] or the fully distributed parameter model described by partial differential equations (PDEs) [20]. Nevertheless, all the aforementioned time domain model based methods adopt frequency independent transmission line models for fault location (the authors did not find any time domain model based fault location algorithm in the existing literature using frequency dependent line models). This is because, if the line model is built in time domain, the frequency dependent models [21-23] usually need to include ODEs or PDEs with time-varying coefficients, which are extremely complicated. On the other hand, if the frequency dependent characteristics are not considered, time domain model based methods will present limited location accuracy especially for UGCs.

This paper proposes a new time domain model based fault location algorithm for UGCs in MMC-HVDC grids considering distributed and frequency dependent parameters. First, a time domain multi-layer model of UGC is proposed, with accurate consideration of distributed and frequency dependent parameters. The model is formulated via a set of PDEs with constant coefficients instead of time-varying coefficients, which greatly simplifies the frequency dependent model. Afterwards, by observing the special asymmetrical structure of the multi-layer model PDEs, a three-step finite difference method is proposed to numerically solve the PDEs and to accurately obtain the line voltage distribution from one terminal. Finally, the fault location is determined via the intersection of calculated voltage distributions. Numerical experiments show that the multi-layer model indeed accurately considers the frequency dependent parameters and results in more accurate line voltage distributions. In addition, the fault location results of the proposed algorithm present less errors compare to those with the existing algorithm. The contributions of the paper are summarized below,

- The paper proposes a multi-layer model in PDEs with constant coefficients; the proposed model accurately describes the physical laws that the UGC obeys, including distributed and frequency dependent parameters; the
The existing classic UGC model is with assumption of frequency independent parameters. However in practice, the parameters of UGCs could vary with different frequencies. Specifically, during severe system transients such as faults, neglecting the frequency dependency of cable parameters could potentially cause modeling errors of transmission lines and thus the fault location errors. This section will first introduce the typical structure of cables, and afterwards present the multi-layer modeling methodology to consider frequency dependent parameters of cables.

### III. UGC Modeling Methodology Using Multi-Layer Model and Partial Differential Equations

In this paper, the two cores of the two cables as well as the soil are separated into \( n_1 \) layers, respectively; the two sheathes and armors of the two cables are separated into \( n_2 \) layers (\( n_2 \) is selected as an odd number for simplification). This separation results in \( 3n_1 + 4n_2 \) layers in the HVDC UGC system. The multi-layer structure is further presented in Figure 3. The core is separated into \( n_1 \) circular layers and \( n_1 - 1 \) annular layers, the sheath, armor, and soil are separated into \( n_2, n_2, \) and \( n_1 \) annular layers. The radius of each layer is defined in the figure. The widths of the layers are selected in such a way that the width of the layer closer to the surface of the conductor is smaller. And for simplification, the width of the outer layer is selected as half of the width of the adjacent inner layer. Here the “surface” of the core is the outer surface; the “surface” of the soil is the inner surface; and the “surfaces” of sheath/armor is both the outer and the inner surfaces. Detailed width selections for the core, the sheath/armor, and the soil are given in (3), (4), and (5), respectively.

**Figure 3. Sketch of the multi-layer structure**
Sheath / Armor: \(2(r_{n+1}\mid n+2 - r_{n+2\mid n+3}) = \ldots = 2^{n-1/2}(r_2 - r_1)\)
\(= (r_{n+2\mid n+3} - r_{n+3\mid n+4}) = 2(r_{n+3\mid n+4} - r_{n+4\mid n+5}) = \ldots = 2^{n-1}(r_{n+1\mid n+2} - r_{n+2\mid n+3})\) (4)

Soil: \((r_{n+1\mid n+2} - r_{n+2\mid n+3}) = 2(r_{n+2\mid n+3} - r_{n+3\mid n+4}) = \ldots = 2^{n-1}(r_2 - r_1)\) (5)

In addition, the multi-layer model treats the two cores as the conductors carrying the two pole currents, while the two sheaths, two armors and the soil are all considered as the ‘ground’. This model adopts the assumption that the voltages of sheath and armors are equal to the voltage of the soil and results in a “three phase” system. Accordingly, the conductor sequence is arranged as positive pole cable core (\(n_1\) layers), negative pole cable core (\(n_1\) layers), positive pole cable sheath and armor (2\(n_2\) layers), negative pole cable sheath and armor (2\(n_2\) layers), and soil (\(n_1\) layers). The layer sequence is arranged from the inner layer to outer layer for each conductor. With the separation of layers, the per unit length parameter matrices of the multi-layer model can be calculated, including the series resistance matrix \(R_l\) (with dimension of \(3n_1+4n_2\)) and series inductance matrix \(L_i\) (with dimension of \(3n_1+4n_2\)), where subscript “p” represents the parameter matrices corresponding to the “layers”. Note that, the shunt capacitance is typically frequency independent, which results in a “phase”-level parameter matrix with dimension of 3. To uniform the notation, the shunt capacitance matrix is also denoted as \(C\). The shunt conductance matrix is ignored. Detail calculations of \(R_l\), \(L_i\) and \(C_i\) are presented in the Appendix.

B. Model of UGCs via Partial Differential Equations

With the layer-level parameter matrices, the multi-layer model of the UGC can be described with a fully distributed parameter circuit, as shown in Figure 4. The circuit is a “three phase” circuit, where the three phases include the “positive phase” (with the positive pole cable core, including \(n_1\) layers), the “negative phase” (with the negative pole cable core, including \(n_1\) layers) and the “ground phase” (with the sheath and armors for the cables, and the soil, including \(n_1+4n_2\) layers).

The line length is \(l\). An infinitesimal line segment at location \(x\) (from terminal \(k\)) is shown in the figure. The line segment length is \(dx\). The phase voltage vector and phase current vector are defined as \(u_p(x,t) = [u_{p1}(x,t) \ u_{p2}(x,t) \ u_{p3}(x,t)]^T\) and \(i_p(x,t) = [i_{p1}(x,t) \ i_{p2}(x,t) \ i_{p3}(x,t)]^T\), respectively. The layer voltage and layer current vector are defined as \(u(x,t) = [u_{11}(x,t) \ u_{12}(x,t) \ \ldots \ u_{1(3n_1+4n_2)}(x,t)]^T\) and \(i(x,t) = [i_{11}(x,t) \ i_{12}(x,t) \ \ldots \ i_{1(3n_1+4n_2)}(x,t)]^T\).

From Figure 4, the physical laws of the circuit is described via the following PDEs in (6),
\[
\partial_t u_p(x,t) + L_i \partial_x \phi_i(x,t) + R_i i_p(x,t) = 0 \quad (6a)
\]
\[
\partial_t i_p(x,t) + C_i \partial_x \phi_i(x,t) + R_i u_p(x,t) = 0 \quad (6b)
\]

One can observe that equation (6) has similar format as (1). Nevertheless, the unknown variables and the corresponding dimensions of the two sub-equations are not the same as (1); the first sub-equation is about the layer voltages \(u_p(x,t)\) and currents \(i_p(x,t)\), with the dimension of \(3n_1+4n_2\) (number of layers), while the second sub-equation is about the phase voltages \(u_p(x,t)\) and currents \(i_p(x,t)\), with the dimension of 3 (number of phases). To solve (6), the relationships between phase and layer voltages/currents should also be considered. By observing that the phase and layer variables share the same nodes (Node \(1\) to Node \(3\) in the Figure (4)), the following equations hold,
\[
u_p(x,t) = E u_p(x,t) \quad (7a)
\]
\[
i_p(x,t) = E^2 i_p(x,t) \quad (7b)
\]
where \(E = [\mathbf{I}_{3n_1\times1} \mathbf{0}_{3n_1\times1} \mathbf{0}_{3n_1\times1} \mathbf{0}_{3n_1\times1} \mathbf{0}_{3n_1\times1} \mathbf{0}_{3n_1\times1} \mathbf{0}_{3n_1\times1}]^T\) is matrix with the dimension of \((3n_1+4n_2)\times3\), and \(L_{occ}\) (or \(\mathbf{0}_{occ}\)) denotes an \(m\)-by-\(n\) matrix with all elements of 1 (or 0).

After substituting (7) into (6), the circuit can be described via the following PDEs in (8), with phase voltages \(u_p(x,t)\) and layer currents \(i_p(x,t)\) as unknown variables,
\[
\partial_t u_p(x,t) + L_i \partial_x \phi_i(x,t) + R_i i_p(x,t) = 0 \quad (8a)
\]
\[
E^2 i_p(x,t) + C_i \partial_x \phi_i(x,t) + R_i u_p(x,t) = 0 \quad (8b)
\]
Note that equation (8) provides a time domain representation of the multi-layer model with constant parameter matrices. If one rewrites (8) into frequency domain, the equivalent resistance and inductance for each phase is actually frequency dependent (detail validations are given in section VI.A). This is exactly why the proposed multi-layer model can well approximate frequency dependent characteristics of the line even with constant parameter matrices in time domain.

IV. PROPOSED NUMERICAL SCHEMES FOR SOLVING MULTI-LAYER MODEL

Next, in order to achieve voltage method fault location, the voltage distribution of the entire line needs to be solved with the multi-layer model in (8) from one terminal of the line. Actually, in the previous work of the authors [20], a method to solve transmission line PDEs in (1) via numerical solutions was proposed. Here similar ideas can also be applied to solve this problem. The numerical solution of PDEs first selects a mesh for the solution area as shown in Figure 5. The distance interval is \(\Delta x\), and the time interval is \(\Delta t\). \(N_t\) and \(N_x\) are the total number of distance and time steps corresponding to the line length and data window. Here the known values include a) initial condition: phase currents, layer currents, and phase voltages at distance step 0 (the local terminal of the line), through entire time steps (the data window); and b) boundary condition: phase currents, layer currents, and phase voltages at time step 0 (the time when/before fault occurs), through entire distance steps (the entire line). The solution direction is along the distance direction, i.e., the solution of all the unknowns at distance step \(j\) are solved with the known values at distance step \(j-1\). The voltage distribution of the entire line is then solved systematically.

Figure 4. Circuit of the multi-layer model
The solution to PDE can be found in Figure 5. Mesh for the numerical solution to the PDE

In fact, according to [20], the solution of PDEs in (1) first eliminates the currents or the voltages in the equation, and transforms the equation into the second order PDEs in (9) with only the voltages or the currents as the unknowns. Afterwards, the voltages or currents are solved with the proposed numerical scheme in [20]. The details of the background of the numerical solution to PDE can be found in [20].

\[
\frac{\partial^2 u(x,t)}{\partial x^2} + B_1 \frac{\partial^2 u(x,t)}{\partial t^2} + B_2 \frac{\partial u(x,t)}{\partial t} + B_3 u(x,t) = 0
\]

where \( B_1 = LC \), \( B_2 = RC + LG \), \( B_3 = RG \), \( C_1 = CL \), \( C_2 = CR + GL \) and \( C_3 = GR \).

However, the problem in (8) cannot be solved directly using the existing procedure in [20]. One can observe that equation (8) is quite “asymmetrical”: the number of unknown voltages is 3 while the number of unknown currents is \( 3n_1 + 4n_2 \); the coefficient matrices before the \( \partial/\partial x \) terms are \( E \) and \( E^T \), which are not invertible. In comparison, for equation (1), the numbers of unknown voltages and currents are both 2; and there are no coefficient matrices (or identity matrices) before the \( \partial/\partial x \) terms. Therefore, if one directly solves (8) with a single numerical scheme such as the procedure in [20], the difference equation corresponding to (8a) has very high redundancy for the solution of phase voltages (there are \( 3n_1 + 4n_2 \) equations and 3 unknown phase voltages; other variables are known from the previous steps). However, the difference equation corresponding to (8b) has infinite number of solutions of the layer currents (there are 3 equations and \( 3n_1 + 4n_2 \) unknown layer currents, other variables are known from the previous steps). These differences determine that the PDE in (8) cannot be rewritten as (9). Therefore, the correct solution cannot be obtained through the existing procedure.

To overcome the limitations, in this paper a three-step method is proposed to directly solve each unknown in the first order PDEs in (8). The three-step method sequentially solves the phase currents, layer currents, and phase voltages at each distance step. To avoid ambiguity, for the rest of the paper, the step in the three-step method is called “step”, while the step in the distance step and time step of the numerical scheme is referred as “distance step” and “time step”, respectively. Next, details of the three-step method are carefully derived. The calculations of the initial and the boundary conditions as well as some discussions of numerical interval and data window are also shown at the end of this section.

A. Step 1: Solution of the Phase Currents

In equation (8b), the number of unknown variables is larger than the number of equations, resulting in infinite number of solutions of the layer currents. However, the number of phase currents is the same as the number of equations. Therefore, with proper formulations, the solution of the phase currents can be unique, where each phase current corresponds to the summation of all the layer currents in each phase. Here the Lax-Wendroff scheme, which is derived in the authors’ previous work [20], is applied to solve the phase currents. The Lax-Wendroff scheme adopts the second order Taylor expansion to approximate \( \partial/\partial x \).

The term \( E^T i_j(x, \Delta x, t) \) can be rewritten as follows,

\[
E^T i_j(x, \Delta x, t) = i_j(x, \Delta x, t) + \Delta x \frac{\partial i_j}{\partial x} + \Delta x^2 \frac{\partial^2 i_j}{\partial x^2} / 2 = \frac{\partial i_j}{\partial x} + \Delta x \frac{\partial i_j}{\partial x} + \Delta x^2 \frac{\partial^2 i_j}{\partial x^2} / 2
\]

where \( i_j \) is the selection of \( i_j \) and \( i_j + \Delta x \), \( R_\text{ave} \) and \( i_\text{ave} \) are matrices with the dimension of \( 3 \times (3n_1 + 4n_2) \), \( i_\text{ave} (k,:) = \sum_{n=1}^{3n_1 + 4n_2} L(k, n) \), \( i_\text{ave} (k,:) = \frac{1}{n} \sum_{i=1}^{N_x} i_\text{ave} (k,:) \), and \( A(k,:) \) denotes the kth row of matrix A. Here \( u_\text{ave} (x,t) = L_\text{ave} i_\text{ave} (x,t) \). Since equation (8a) has very high redundancy (3n_1+4n_2 equations and 3 unknown phase voltages), the average of columns of the coefficient matrices is considered, to minimize the numerical error.

Rewrites equation (10) into the difference form to solve the phase currents (\( j = 0, 1, ..., N_x-1 \)),

\[
I_j = I_j + \Delta t \left( -C_j (U_j^{-1} - U_j) / (2 \Delta t) \right) + \Delta t^2 / 2 \cdot C_j L_\text{ave}
\]

where the uppercase letters denote the numerical solution of the corresponding variable. The superscript \( n \) and subscript \( j \) denote the time and distance step number, respectively. For instance, the numerical solution corresponding to \( i_j(x, \Delta x, \Delta t) \) is represented as \( I_j^{(n)} \), \( j = 0, 1, ..., N_x \) and \( n = 0, 1, ..., N_t \).

In addition, the Lax-Wendroff scheme requires a certain condition on the selection of \( \Delta x \) and \( \Delta t \) for stable solutions. From the authors’ previous conclusion in [20], here similarly the CFL condition could be considered as the stable condition, i.e., \( \Delta x / \Delta t \) should be less than the smallest wave velocity of all the line modes. For the cable system, the smallest wave velocity is typically larger than 1.2×10^8 m/s. To provide some safe margins, here the selection of \( \Delta x / \Delta t = 1 \times 10^8 \) m/s could normally satisfy the stable condition.

To sum up, from (11), the first step of the proposed three-step scheme utilizes the phase currents, layer currents, and phase voltages at distance step \( j \), to solve the phase currents at distance step \( j + 1 \). The first step of the three-step scheme with \( j = 0 \) as an example is shown in Figure 6 (a).

B. Step 2: Solution of the Layer Currents

With the solved phase currents \( i_j(x, t) \) at distance step \( j + 1 \) from section IV.A, the layer currents can be obtained. The following relationship holds between the layer currents and the phase currents,

\[
E^T i_j(x, t) = \Delta x \frac{\partial i_j}{\partial x} + \Delta x^2 \frac{\partial^2 i_j}{\partial x^2} / 2
\]

One can observe from (12) that there are 3 equations and \( 3n_1 + 4n_2 \) unknown layer currents. Therefore, \( 3n_1 + 4n_2 - 3 \)
additional equations are required to solve the layer currents. In fact, these $3n_l+4n_t-3$ additional equations can be derived from (8a). From the structure of matrix $E$, the rows of (8a) can be represented as,

$$
\tilde{c}_k(x, t) = -L_i(k)\tilde{c}_k(x, t) \tilde{c}_t - R_i(k)\tilde{c}_k(x, t), \quad k = 1, \ldots, n_l
$$

$$
\tilde{c}_u_i(x, t) = -L_i(k)\tilde{c}_u_i(x, t) \tilde{c}_t - R_i(k)\tilde{c}_u_i(x, t), \quad k = 1+n_l, \ldots, 2n_l \quad (13)
$$

$$
\tilde{c}_u_j(x, t) = -L_i(k)\tilde{c}_u_j(x, t) \tilde{c}_t - R_i(k)\tilde{c}_u_j(x, t), \quad k = 1+2n_l, \ldots, 3n_l+4n_t
$$

One can observe that there are $3n_l+4n_t$ equations in (13). Note that the values of $\tilde{c}_u_i/\tilde{c}_x$, $\tilde{c}_u_i/\tilde{c}_x$ and $\tilde{c}_u_i/\tilde{c}_x$ are not available. To eliminate those variables, for each of the three sub-equations, one can simply subtract one row from another, resulting in $n_l-1$, $n_l-1$ and $n_l+4n_t-1$ independent equations, i.e.,

$$
L_{adg}(\tilde{c}_k(x, t) \tilde{c}_t + R_{adg}(x, t)) = 0
$$

(14)

where $L_{adg}$ is (similar relationship holds between $R_{adg}$ and $R_{adg}$), $L_{adg}(k, :) = [L_i(1, :) - L_i(k, :)], k = 1, \ldots, n_l-1$

$$
L_{adg}(k, :) = [L_i(1+n_l, :) - L_i(k-1+n_l, :)], k = 1, \ldots, n_l-1
$$

(15)

$$
L_{adg}(k, :) = [L_i(1+2n_l, :) - L_i(k-1+2n_l, :)], k = 1, \ldots, n_l+4n_t-1
$$

With equations (12) and (14), there are exactly $3n_l+4n_t$ equations to solve $3n_l+4n_t$ layer currents. In fact, equation (12) is a set of algebraic equation, and equation (14) is a set of ordinary differential equations (ODEs). Rewrite (14) into the difference form using trapezoidal integration method (which is an unconditionally stable scheme) [27].

$$
L_{adg}(i, t)-i(t-(\Delta t)) + R_{adg}(i, t) \cdot (\Delta t/2) = 0
$$

(16)

Afterwards, the solution of the layer currents can be obtained with (12) and (16) ($j = 0, \ldots, N_t-1; n = 1, \ldots, N_l-1$),

$$
I_{j+1} = \frac{L_{adg}([\Delta t/2] + R_{adg})}{E^T} \left[ L_{adg}([\Delta t/2] - R_{adg}) I_{j+1} \right]
$$

(17)

To sum up, from (17), the second step of the proposed three-step scheme utilizes $a$ the phase currents at distance step $j+1$ (solved from the first step of the three-step scheme), and $b$ the layer currents at time step 0, distance step $j+1$, to solve the layer currents at distance step $j+1$. The second step of the three-step scheme with $j = 0$ as an example is shown in Figure 6 (b).

C. Step 3: Solution of the Phase Voltages

With the solved layer currents $i_j(x, t)$ at distance step $j+1$ from section IV.B, the phase voltages can be obtained. Equation (8a) has very high redundancy: $3n_l+4n_t$ equations and only 3 unknown phase voltages. Therefore, the average of the columns of the coefficient matrices are considered. Equation (8a) can be rewritten as,

$$
\tilde{c}_u_i/\tilde{c}_x + L_{av}(x) \tilde{c}_u_j/\tilde{c}_x + R_{av}i_j = 0
$$

(18)

where $L_{av}$ and $R_{av}$ are defined in (10).

For equation (18), the Lax-Wendroff scheme in [20] cannot be applied since the second order derivative of the phase voltages cannot be explicitly obtained (if one directly follows the idea in equation (10), after expanding $u_p(x, t)$, the term $\Delta t^2/2u_{p,t}(x, t)\Delta t^2/2(-L_{av}\tilde{c}_u_i(x, t) - R_{av}i_j(x, t))$ cannot be represented by functions of $u_p(x, t)$ since the matrix $E^T$ in equation (8b) is not invertible). Here the implicit scheme is applied to solve (18) (the scheme is unconditionally stable) [28],

$$
U_{p,j+1} - U_{p,j} = \Delta t L_{av}(I_{j+1} - I_{j-1})/(2\Delta t) - \Delta t R_{av} I_{j+1} = 0
$$

(19)

Note that, here the unknown phase voltages have already been solved in step 2 from section IV.B. Therefore, the phase voltages are solved with explicit formulations ($j = 0, \ldots, N_t-1; n = 1, \ldots, N_l-1$),

$$
U_{p,j+1} - U_{p,j} = -\Delta t L_{av}(I_{j+1} - I_{j-1})/(2\Delta t) - \Delta t R_{av} I_{j+1} = 0
$$

(20)

and ($j = 0, \ldots, N_t-1; n = 1, \ldots, N_l-1$),

$$
U_{p,j+1} - U_{p,j} = -\Delta t L_{av}(I_{j+1} - I_{j-1})/(2\Delta t) - \Delta t R_{av} I_{j+1} = 0
$$

(21)

To sum up, from (20) and (21), the third step of the proposed three-step scheme utilizes $a$ the phase voltages at distance step $j$, and $b$ the layer currents at distance step $j+1$ (solved from the second step of the three-step scheme), to solve the phase voltages at distance step $j+1$. The third step of the three-step scheme with $j = 0$ as an example is shown in Figure 6 (c).

Until now, the voltage distribution along entire line (i.e. the phase voltages at each distance step) is solved systematically.

D. Calculation of Initial and Boundary Conditions

The initial condition includes phase currents, layer currents, and phase voltages at distance step 0 through the entire time steps. The positive and negative phase currents and voltages can be directly obtained from terminal instantaneous measurements. From Kirchhoff’s Current Laws (KCLs), the ground phase current at terminal should be equal to the negative of the summation of the positive phase and negative phase currents for every time step. The ground phase voltage is assumed to be zero at both line terminals. For the layer currents, at time step 0 (the starting time, which is before the occurrence of the fault), the current inside the line is close to DC. Therefore, the current for each layer is proportional to the reciprocal of the resistance as,

$$
i_j(x, 0) = R_{j-1}^{-1}ER_j i_j(x, 0)
$$

(22)

where $x = 0 \cdot R_p = \{R_0, 0, 0, 0, R_2, 0, 0, 0, R_1\}$, $R_0 = 1/\sum_{k=0}^n(1/R_{ik}), R_2 = 1/\sum_{n=1}^2(1/R_{ik}), R_1 = 1/\sum_{k=2n+1}^{2n+4}(1/R_{ik})$.

Afterwards, the layer currents $i_j(0, t)$ (at each time step and distance step 0) can be calculated with (17) and phase currents at distance step 0.

The boundary condition includes phase currents, layer
currents, and phase voltages at time step 0, through entire distance steps. At time step 0, since the system is before the occurrence of the fault, the voltages and the currents are assumed to be linearly distributed,
\[ i_p(x, 0) = \frac{x}{l} \cdot i(l, 0) \]
\[ i_j(x, 0) = \frac{x}{l} \cdot i(l, 0) + \frac{l - x}{l} \cdot i(l, 0) \]
\[ u_p(x, 0) = \frac{x}{l} \cdot u(l, 0) \]
\[ u_j(x, 0) = \frac{x}{l} \cdot u(l, 0) + \frac{l - x}{l} \cdot u(l, 0) \]
where \( i_p(0, 0) \) and \( i_j(l, 0) \) are obtained according to (22).

E. Selection of Numerical Intervals and Data Window

The numerical intervals should be selected to ensure stability of the proposed three-step method. Since the schemes in the second and third steps of the proposed three-step method are unconditionally stable, once \( \Delta t/\Delta x = 1 \times 10^8 \) m/s is satisfied for the scheme in first step (as shown in section IV.A), the entire three-step method is stable.

The data window should be selected to ensure that the calculated line voltage distribution covers the full length of the line. In fact, from the mesh for the numerical solution as shown in Figure 5, one can observe that the last time step of the solved variables is decreased by 1 if the distance step is increased by 1 (the last time steps are \( N_i, N_i - 1, N_i - 2, \ldots, N_i - N \) with distance steps 0, 1, 2, ..., \( N_i \)) [20]. Therefore, \( N_i \) should be selected such that \( N_i > N \), where \( N_i \Delta t \) is the length of the data window; in this case, the line voltage distribution is available within the time steps of \([0, N_i - N_i] \).

It is also worth noting that the calculation procedure introduced in section IV.C calculates the voltage distribution from terminal k as an example. The solution of the voltage distribution from terminal m is similar.

V. FAULT LOCATION ALGORITHM

The essence of the voltage method fault location is that the intersection of the voltage distributions calculated from two terminals respectively indicates the fault location. The following optimization problem is formulated for fault location to guarantee the accuracy,
\[ \min F(x) = \frac{1}{N_i - N} \sum_{i=0}^{N} f(x, t) \]
where \( f(x, t) \) is defined as the absolute difference between the voltage distributions calculated from terminal k and those calculated from terminal m, i.e.
\[ f(x, t) = |u^{(k)}(x, t) - u^{(m)}(x, t)|. \]
Here \( u^{(k)}(x, t) = u_k^{(0)}(x, t) + u_k^{(i)}(x, t) \) (\( j = k, m \)), to cover different fault types. The summation window is \([0, t_f] \), where \( t_f = (N_i - N) \Delta t \) (which is consistent with section IV.E).

In fact, the optimization problem can be directly solved by first calculating all the values of \( F(x) \) corresponding to different fault locations \( x \) (with distance interval of \( \Delta x \)), and then selecting the fault location with minimum \( F(x) \). It can be observed that the calculation burden is proportional to \( 1/\Delta x \) (since the \( N_i - N \) is proportional to \( 1/\Delta x \) with certain line length and data window), i.e. the calculation burden could be high if a small \( \Delta x \) is adopted to minimize the fault location error. Therefore, here a two-iteration algorithm is adopted [20]. The first iteration calculates the approximate fault location \( x_{\text{iter1}} \) with relatively large numerical intervals \( \Delta x_{\text{iter1}} \) and \( \Delta t_{\text{iter1}} \), within the length of the entire line \([0, l] \). The second iteration fine-tunes the accurate fault location \( x_{\text{iter2}} \) with smaller numerical intervals \( \Delta x_{\text{iter2}} \) and \( \Delta t_{\text{iter2}} \), within the line segment \([x_{\text{iter1}} - \Delta x_{\text{iter1}}, x_{\text{iter1}} + \Delta x_{\text{iter1}}] \). Through this procedure, the computational complexity is greatly alleviated.

VI. NUMERICAL EXPERIMENTS

The example test system is constructed in PSCAD/EMTDC. The system is a ±320 kV two-pole MMC-HVDC grid, as shown in Figure 7. The UGC of interest is UGC k-m with the length of 200 km, and its structure is also shown in the figure. The cable is built using the frequency dependent (phase) model in PSCAD/EMTDC. The resistivity and permittivity of the conductors and insulators are shown in Table 1. The available measurements are the two-pole instantaneous voltages and currents at both terminals of k-m, with the sampling rate of 20 kHz and the data window of 5 ms after the fault occurs. A 2nd order Butterworth anti-aliasing filter with the cut-off frequency of 5 kHz is applied before sampling. The intervals of the numerical scheme are selected according to section IV.E, as \( \Delta x_{\text{iter1}} = 1 \) km, \( \Delta t_{\text{iter1}} = 10 \) ms, \( \Delta x_{\text{iter2}} = 100 \) m, \( \Delta t_{\text{iter2}} = 1 \) ms.

Here the cubic spline interpolation is adopted to complete the dataset, as the sampling interval is larger than the numerical time interval.

![Image](a) UGC system (b) Cable structure

![Figure 7. Test system with (a) the UGC system and (b) cable structure](https://example.com/image7.png)

**Table 1. Coefficients of resistivity and permittivity**

<table>
<thead>
<tr>
<th>Conductor</th>
<th>Resistivity (Ω·m)</th>
<th>Insulator</th>
<th>Permittivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core</td>
<td>( \rho_1 = 1.68 \times 10^{-8} )</td>
<td>Insulator 1</td>
<td>( \epsilon_1 = 2.3\epsilon_0 )</td>
</tr>
<tr>
<td>Sheath</td>
<td>( \rho_2 = 2.20 \times 10^{-7} )</td>
<td>Insulator 2</td>
<td>( \epsilon_2 = 2.3\epsilon_0 )</td>
</tr>
<tr>
<td>Armor</td>
<td>( \rho_3 = 1.80 \times 10^{-7} )</td>
<td>Insulator 3</td>
<td>( \epsilon_3 = 2.3\epsilon_0 )</td>
</tr>
<tr>
<td>Soil</td>
<td>( \rho_s = 100.00 )</td>
<td>( \epsilon_s = 8.85 \times 10^{-12} ) F/m</td>
<td></td>
</tr>
</tbody>
</table>

In the rest of this section, the frequency dependent characteristic of the multi-layer model is firstly validated. Next, a specific positive pole to ground fault is taken as an example to demonstrate the proposed fault location procedure. Afterwards, the performance of the proposed algorithm is further verified via various faults with different fault types, locations and resistances. The performances of the existing Bergeron model based voltage method [18] are also provided, to demonstrate the importance of considering frequency dependent parameters during UGC fault location. Note that the existing method also locates the fault via the two-iteration algorithm in section V, but the voltage distribution is solved using the Bergeron model in (2), which assumes frequency independent parameters. Here the fault location schemes are realized in Matlab R2016a.

A. Validating Frequency Dependency of the Multi-Layer Model

In order to numerically examine whether the multi-layer
model can accurately represent the frequency dependent parameters, the equivalent impedance of the multi-layer model at each frequency \( f \) is calculated. Note that, since the modeling of the line shunt capacitances is frequency independent (as explained in the Appendix), only the frequency dependency of the line series resistance and series inductance are validated. Rewrite (8a) in frequency domain (where \( \tilde{x} \) corresponding to the phasor representation of variable \( x \),

\[
E \frac{d\tilde{U}_p(x)}{dx} + Z_{eq} \tilde{I}_p(x) = 0
\]

(25)

where \( Z_i = R_i + j\omega L_i \), and \( \omega = 2\pi f \) is the angular frequency.

From (7b), the phase currents \( \tilde{I}_p(x) \) is equal to \( E^T \tilde{I}_p(x) \), where the ground phase current equals to negative of the summation of the positive phase and negative phase currents. Therefore, pre-multiply the matrix \( E^T Z_{eq}^{-1} \) to (25), replace \( E^T \tilde{I}_p(x) \) with \( \tilde{I}_p(x) \) and eliminate the ground phase, the solution for \( dU(x)/dx \) is,

\[
dU(x)/dx = -Z_{eq} \tilde{I}_p(x)
\]

(26)

where \( U(x) \) and \( I(x) \) are the two pole voltage and current phasor vectors corresponding to (1a), the elements in equivalent impedance matrix \( Z_{eq} \) are \( Z_{eq}(i,j) = Z_{eq,\text{withground}}(i,j) - Z_{eq,\text{withground}}(i,3) \) \( (i,j = 1, 2) \). \( Z_{eq,\text{withground}} = (E^T Z_{eq}^{-1} E)^{-1} \), \( A(i,j) \) denotes the element at the \( j \)th row and \( j \)th column of matrix \( A \).

Afterwards, the equivalent series resistances and inductances can be calculated, where the resistances are the real part of the impedances, and the inductances are the imaginary part of the impedances divided by the angular frequency \( \omega \). The equivalent series resistances and inductances are both functions of \( \omega \).

Next, the equivalent series resistances and inductances of the proposed multi-layer model are numerically calculated and compared to those of the frequency independent model in (1) and the actual frequency dependent model (FDM). Here the actual frequency dependent parameters of the cables are obtained from PSCAD/EMTDC. Select the layer number of the multi-layer model as \( n = 1, 3, 5, 10 \), respectively. The self and mutual equivalent resistances and inductances are shown in Figures 8 (a1) to (d1). Similarly, select the layer number of the multi-layer model as \( n = 3 \) and \( n = 5, 7 \), respectively. The self and mutual equivalent resistances and inductances are not specified in the figures, since the two pole cables have identical structures and identical impedances. One can observe that the multi-layer model indeed approximates the frequency dependent parameters much more accurate than the frequency independent model with constant parameters. Additionally, from Figures 8 (a1) to (d1), the accuracy increases with larger number of \( n \) (the results with \( n = 5 \) is more accurate than those with \( n = 3 \), but the results with \( n = 5 \) or 10 are almost on top of each other). From Figures 8 (a2) to (d2), the accuracy are approximately the same with different number of \( n \) (the results with \( n = 3, 5 \) or 7 are almost on top of each other). Therefore, the layer numbers are selected as \( n = 5 \) and \( n = 3 \) for the rest of the paper, to ensure not only adequate modeling accuracy but also minimum calculation complexity (numerical experiments have also verified that the fault location results with the selection of \( n = 10 \) and \( n = 5 \) are almost the same as those with the selection of \( n = 5 \) and \( n = 3 \)). In addition, it can also be observed that there are still some parameter gaps between the multi-layer model and the FDM, especially for resistances. This is actually caused by the adopted assumptions of the multi-layer model (as shown in Appendix). Nevertheless, with the proposed multi-layer model, the solutions of the voltage distributions as well as the fault location results are still with sufficient accuracy, as shown in the following sections.

B. Demonstrating Fault Location Results with an Example Fault

Next, a P (positive pole) to G (ground) fault is selected as an example to demonstrate the two-iteration fault location procedure. The fault resistance is 0.01 \( \Omega \), and the actual fault location is 60 km from terminal k. The fault occurs at 0.7 s. The
available data window for fault location after the occurrence of the fault is 5 ms. Two pole voltage and current measurements at two terminals of the line are shown in Figure 9. The frequency spectrums of the measurements extracted by FFT are shown in Figure 10. One can observe that the majority of the frequency components lies within the frequency band of [0, 2000] Hz. This frequency band indeed corresponds to the frequency dependent characteristics of the transmission line parameters. Based on Figure 8, the proposed method accurately considers the frequency dependency within this frequency band. Next, the detailed two-iteration fault location algorithm is further explained.

In iteration 1 of the fault location algorithm, Figures 11 (a1) and (b1) depict the absolute difference between the voltage distributions calculated from terminals k and m (i.e. $f(x, t)$ in (24)), for the existing and the proposed algorithms, respectively. With $\Delta x_{iter1} = 1$ km, $\Delta N_{iter1} = 10$ µs, the data window $N_1 \Delta x_{iter1} = 5$ ms and the line length $N_1 \Delta x_{iter1} = 200$ km, $N_1 \cdot N_1$ is equal to 300. Therefore, according to (24), the voltage from time step 0 to step $(N_1 - N_1) = 300$, corresponding to time 0 to 3 ms, is solved. If the calculated voltage distribution is accurate, the minimum of the voltage difference should always correspond to the actual fault location, regardless of time. From Figure 11 (a1), for the existing algorithm, the minimum of the voltage distribution varies dramatically (the “valley” of the “mountain” is a meandering curve), since the voltage calculation fails to consider frequency dependency of the UGC parameters. On the other hand, from Figure 11 (a2), for the proposed algorithm, with the accurate modeling of the frequency dependent parameters, the minimum of the voltage distribution is around 60 km at every time step (the “valley” of the “mountain” is basically a straight line in parallel with the time axis), implying that the calculated voltage distribution is accurate. According to (24), in iteration 1, the fault locations are determined as $x_{iter1} = 55$ and 60 km for the existing and the proposed algorithms, respectively, as shown in Figures 11 (b1) and (b2).

In iteration 2, the fault location is fine-tuned within the range of $[x_{iter1} - \Delta x_{iter1}, x_{iter1} + \Delta x_{iter1}]$. Similarly, the voltage differences for the existing and the proposed algorithms are shown in Figures 11 (c1) and (c2), respectively. The fault location results according to (24) are $x_{iter2} = 55.6$ and 59.8 km for the existing and the proposed algorithms, respectively, as shown in Figures 11 (b1) and (b2). This indicates that for this example fault event, the proposed multi-layer model based fault location algorithm presents less error than the existing frequency independent model based algorithm.

C. Fault Location Results with Various Faults

In this section, the validity of the proposed algorithm is further examined. Several groups of faults are tested, including low resistance single pole to ground faults, low resistance
double pole to ground faults, and high resistance faults. Here the pole to pole faults are not considered since the system setup of two individual cables (corresponding to two poles) will hardly result in pole to pole faults without ground. The actual fault location is selected as every 20 km through the line. Equation (27) defines the absolute fault location (FL) error. For each case, the fault location results of the proposed algorithm are compared to those of the existing algorithm. The fault location procedure for each event is same as those shown in Section VI. B, including the numerical intervals, data window and summation window. Extensive experiments verify that the calculation time of the proposed algorithm is always less than 10 seconds even without parallel computing techniques (with CPU: Intel i7-7700). Since fault location is an offline procedure, the calculation burden is acceptable for practical implementations.

\[
\text{Abs FL Error} = \frac{\text{Estimated FL} - \text{Actual FL}}{\text{Total Length of the Line}} \times 100\% \quad (27)
\]

1) Low Resistance Single Pole to Ground Faults

Figure 12 (a) shows the performances of the existing and the proposed fault location algorithms of low resistance (0.01, 1 and 10 Ω) P (positive pole) to G (ground) faults. The detailed values of the fault location errors are summarized in the first 3 rows of Table 2.

2) Low Resistance Double Pole to Ground Faults

Figure 12 (b) shows the performances of the existing and the proposed fault location algorithms of low resistance (0.01, 1 and 10 Ω) P and N (positive and negative poles) to G faults. The detailed values of the fault location errors are summarized in the next 3 rows of Table 2.

3) High Resistance Faults

Figure 12 (c) shows the performances of the existing and the proposed fault location algorithms of high resistance (50, 100 and 200 Ω) P to G faults. The detailed values of the fault location errors are summarized in the last 3 rows of Table 2.

To sum up, for different fault types, fault locations and fault resistances, the proposed fault location algorithm has less error than the existing algorithm.

VII. DISCUSSION

This section discusses the factors that can potentially influence the performances of the proposed fault location algorithm. Also, some other discussions are carried out in this section.

A. Influence of the Measurement Error

The measurement errors obeyed the Gaussian distribution are added to the available voltage and current instantaneous measurements. The standard deviations of the measurement errors are 1%, 2% and 5%, respectively. Figure 13 (a) shows the performances of the existing and the proposed fault location algorithms with different measurement errors. The detailed values of the fault location errors are summarized in the first 3 rows of Table 3. The proposed algorithm demonstrates robustness towards measurement errors.

Table 3. Absolute fault location errors with different measurement errors

<table>
<thead>
<tr>
<th>Factors</th>
<th>Values</th>
<th>Average error (%)</th>
<th>Maximum error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement error</td>
<td>1 (%)</td>
<td>1.76</td>
<td>3.10</td>
</tr>
<tr>
<td></td>
<td>2 (%)</td>
<td>1.83</td>
<td>3.60</td>
</tr>
<tr>
<td></td>
<td>5 (%)</td>
<td>2.32</td>
<td>6.75</td>
</tr>
<tr>
<td>Parameter error</td>
<td>1 (%)</td>
<td>1.99</td>
<td>3.80</td>
</tr>
<tr>
<td></td>
<td>2 (%)</td>
<td>2.00</td>
<td>3.90</td>
</tr>
<tr>
<td></td>
<td>5 (%)</td>
<td>2.25</td>
<td>4.75</td>
</tr>
<tr>
<td>Sampling rate</td>
<td>20 (kHz)</td>
<td>1.87</td>
<td>3.60</td>
</tr>
<tr>
<td></td>
<td>50 (kHz)</td>
<td>2.08</td>
<td>3.60</td>
</tr>
<tr>
<td></td>
<td>100 (kHz)</td>
<td>1.80</td>
<td>2.80</td>
</tr>
</tbody>
</table>

B. Influence of the Parameter Error

The parameters for the multi-layer model are calculated according to the Appendix with certain values of resistivity and permittivity as shown in Table 1. However, such values may not be very accurate in practice, and could be influenced by the weather, humidity, cable usage time, etc. Here the 1%, 2% and 5% errors are added to all these values respectively in both the existing and the proposed fault location algorithm. Figure 13 (b) shows the performances of the existing and the proposed fault location algorithms with different parameter errors. The detailed values of the fault location errors are summarized in the next 3 rows of Table 3. The proposed algorithm demonstrates robustness towards parameter errors.
Influence of the sampling rate during field implementations. Since the ions in MMC lines, it is worth noting that protection (also known as protection problem) is an online procedure and determines whether there is an internal fault occurs within the line under protection. It focuses on the protection speed (the DC circuit breaker should trip the line within several milliseconds after the occurrence of the fault) as well as the protection reliability (dependably trip internal faults and securely ignore external faults), and typically does not require to determine the exact fault location within the line [29, 30]. On the contrary, the fault location problem is an offline procedure and determines the exact fault location within the faulted line, after the line is tripped by the DC circuit breaker (knowing that there is an internal fault). It typically utilizes the voltage and current measurements during the fault before the faulted line is isolated (with short data window of several milliseconds for MMC-HVDC grids) since they contain rich fault location information. The fault location problem focuses on the fault location accuracy rather than the calculation speed; the calculation time of several minutes is acceptable in practice.

This paper focuses on the fault location problem. Among existing fault location methods, traveling wave based fault location methods are well known. However, traveling wave based fault location methods encounter limitations in MMC-HVDC UGCs, including wavefront detection reliability issues (due to high fault resistance and frequency dependent characteristics of UGCs) and high sampling rates. The fault location method proposed in this paper overcomes those limitations of traveling wave based methods: it does not require reliable detection of wavefronts, and the fault location can be accurately determined with a relatively low sampling rate of 20 kHz.

Finally, the proposed fault location method may still encounter challenges during field implementations. Since the proposed method is a model based fault location method, the accuracy of the fault location results is dependent on the accuracy of transmission line model. Specifically, the modeling of underground cables require accurate geometric parameters (cable radii, distance between cables, etc.) and material parameters (conductor resistivity, soil resistivity, insulator permittivity, etc.), some of which may vary due to degradation of cables and different weather conditions. In this case, online parameter identification approaches or parameter-free fault location methods could benefit the fault location results. In addition, the validation of the proposed method using field data will be extremely valuable. Those issues will be studied in future publications.

C. Influence of the Sampling Rate

The sampling rates are adopted as 20, 50 and 100 kHz, respectively. Figure 13 (c) shows the performances of the existing and the proposed fault location algorithms with different sampling rates. The detailed values of the fault location errors are summarized in the last 3 rows of Table 3. From the numerical validation, the fault location results of the proposed algorithm are sufficiently accurate with the sampling rate of 20 kHz; the higher sampling rates do not improve much the fault location accuracy.

D. Other Discussions

In fact, the proposed fault location methodology is similarly applicable to other HVDC (LCC-HVDC, two/three-level VSC-HVDC, MMC-HVDC, etc.) or HVAC underground cables, since the methodology is based on accurate modeling of underground cables and does not have specific assumptions on the power sources (AC/DC) or the control methods of the rest of the system. In addition, if one would like to locate faults within overhead transmission lines considering distributed and frequency dependent parameters, similar methodology could also be applied (except that the parameters of the multi-layer model in the Appendix require re-derivation for overhead lines).

In this paper, the proposed multi-layer model enabled fault location algorithm is applied to the MMC-HVDC grids system with underground cables. It is because this scenario is one of the challenging situations where the distributed and frequency dependent parameters have high impact on the accuracy of the fault location results, and the available time window during faults is usually quite short. The results prove that the proposed fault location methodology accurately considers distributed and frequency dependent parameters, and presents higher fault location accuracy compared to the existing method.

Besides, it is worth noting that protection (also known as protective relaying) and fault location on MMC-HVDC lines are two different problems. The protection problem is an online procedure and determines whether there is an internal fault occurs within the line under protection. It focuses on the protection speed (the DC circuit breaker should trip the line within several milliseconds after the occurrence of the fault) as well as the protection reliability (dependably trip internal faults and securely ignore external faults), and typically does not require to determine the exact fault location within the line [29, 30]. On the contrary, the fault location problem is an offline procedure and determines the exact fault location within the faulted line, after the line is tripped by the DC circuit breaker (knowing that there is an internal fault). It typically utilizes the
validate that, the frequency dependent characteristics of UGCs can be accurately represented via the proposed multi-layer model. Also, the results of the proposed fault location algorithm are more accurate than those of the existing Bergeron model based algorithm (with frequency independent parameters). In addition, the proposed fault location algorithm presents sufficient accuracy with 5 ms data window and 20 kHz sampling rate, regardless of fault type, fault location and fault resistance.

REFERENCES


APPENDIX

This Appendix presents the calculation of per unit length parameter matrices of the multi-layer model, including the series resistance matrix $R_s$, series inductance matrix $L_s$, and the shunt capacitance matrices $C_s$. These three matrices are all square matrices. This paper adopts the “physical derivation” to clarify the physical meaning of each parameter. The derivation holds following assumptions: 1) for each layer, the current is uniformly distributed within the layer, respectively; 2) The entire coaxial annular soil ring is assumed to surround each cable, respectively; and 3) The radius of the cable is much smaller than the distance between two cables.

### A. Line Series Resistance Matrix $R_s$

The matrix $R_s$ is with the dimension of $3n_1+4n_2$. The diagonal elements of $R_s$ correspond to the self-resistances of the layers. Note that there are only two types of layer structures: circular layer and annular layer. The self-resistance for a circular layer with radius $r$ is shown in (A-1), while the self-resistance for an annular layer with inner radius $r_i$ and outer radius $r_o$ is given in (A-2). From the physical structure of the multi-layer model, the off-diagonal elements of $R_s$ (mutual-resistance between two different layers) are zero.

\[
R_{\text{circular}} = \rho/(2\pi r^2) \quad (A-1)
\]

\[
R_{\text{annular}} = \rho/[\pi(r_o^2 - r_i^2)] \quad (A-2)
\]

### B. Line Series Inductance Matrix $L_s$

The matrix $L_s$ is also with the dimension of $3n_1+4n_2$. The inductance matrix can be derived according to the Ampère’s...
circuitual law. The values of self-inductances are also separated into circular layer (radius \( r \)) and annular layer (inner and outer radius \( r_i \) and \( r_o \)), as given in (A-3) and (A-4), respectively,

\[ L_{\text{circular}} = \mu_0 \mu / (2 \pi) \cdot \left[ \frac{1}{4} + \ln(1/r) \right] \quad \text{(A-3)} \]

\[ L_{\text{annular}} = \mu_0 \mu / (2 \pi) \cdot \left[ \frac{(r_o^4 - r_i^4)}{4} - r_i^2(r_o^3 - r_i^3) + r_i^4 \log(r_o/r_i) \right] \quad \text{(A-4)} \]

where \( \mu_0 \) is the permeability of vacuum, \( \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \).

The calculation of the mutual-inductance is separated into the mutual-inductances between two concentric layers (two layers that share the same center, including: both from one cable, both from the soil, one from the cable and another from the soil) and the mutual-inductances between layers with different centers (two layers from two different cables).

Consider the mutual-inductance between two concentric layers. The two layers always include an annular layer as the outer layer. The mutual-inductance between such two layers is only dependent on the structure of the outer annular layer (with inner and outer radius \( r_i \) and \( r_o \)),

\[ M = \mu_0 \mu / (2 \pi) \cdot \left[ \left( \frac{(r_o^2 - r_i^2)}{2} - r_i^2 \log(r_o/r_i) \right) / (r_o^3 - r_i^3) + \ln(1/r_i) \right] \quad \text{(A-5)} \]

Consider the mutual-inductance between two layers from two different cables. With the assumption that the radius of the cable is much smaller than the distance \( d \) between two cables, the mutual-inductance is only dependent on the distance \( d \),

\[ M_i = \mu_0 \mu / (2 \pi) \cdot \ln(1/d) \quad \text{(A-6)} \]

C. Line Shunt Capacitance Matrix \( \mathbf{C} \)

Different from the \( \mathbf{R} \) and \( \mathbf{L} \) matrices, the matrix \( \mathbf{C} \) is with the dimension of 3 (as mentioned in section III.A). For underground cables, the sheath, armor and soil demonstrate electrostatic shielding to the inner conductors. In addition, here the sheath, armor and the soil are considered together as one “ground phase” (as mentioned in section III.A and B). Therefore, the capacitors only exist between the most outer surface of the “ground phase” (i.e. the inner surface of sheath) and the most outer surface of the “positive/negative phase” (i.e. the outer surface of core). The capacitance can be derived according to the Gauss’ law,

\[ C = 2 \pi e_0 / \log(r_z/r_i) \quad \text{(A-7)} \]

where \( e_1 \), \( r_1 \) and \( r_2 \) are defined in Figure 2. Here the model of the capacitance is with frequency independent parameters.

Afterwards, according to Figure 4, the capacitance matrix (here the structures and permittivities of cables are assumed the same; the formulation is similar for cables with different structures or permittivities),

\[
\mathbf{C} = \begin{bmatrix}
C & -C & \\
- C & C & 2C \\
- C & - C & 2 C
\end{bmatrix} \quad \text{(A-8)}
\]