Transmission Line Fault Location in MMC-HVDC Grids Based on Dynamic State Estimation and Gradient Descent

Binglin Wang, Student Member, IEEE, Yu Liu, Member, IEEE, Dayou Lu, Student Member, IEEE, Kang Yue, Student Member, IEEE, and Rui Fan, Member, IEEE

Abstract—This paper proposes a new fault location method for transmission lines in MMC-HVDC grids based on dynamic state estimation (DSE) and gradient descent. The method only requires a short data window of 5 ms after the occurrence of the fault and therefore is applicable for MMC-HVDC grids with high-speed tripping techniques. The method first builds a high-fidelity linear dynamic model of the DC transmission line, which accurately describes physical laws of the transmission line during the fault. Afterwards, the consistency between the measurements and the linear dynamic model is evaluated through the DSE algorithm. Finally, the actual fault location which corresponds to the best consistency is determined via the gradient descent algorithm. Compared to the existing DSE based fault location methods which solve highly nonlinear DSE problems, the proposed method only needs to solve a series of linear DSE problems, which overcomes the issues such as large numerical error and high computational burden especially for transmission lines in MMC-HVDC grids. Numerical experiments validate the effectiveness of the proposed method, with different fault types, resistances and locations. In addition, the method only requires a relatively low sampling rate of 20k samples per second.

Index Terms—Fault location, transmission lines, MMC-HVDC grids, dynamic state estimation, gradient descent.

I. INTRODUCTION

HIGH voltage direct current (HVDC) power transmission systems are widely adopted in modern power grid to efficiently transmit large amount of power over long distances. The modular multilevel converter (MMC) topology in HVDC converter stations is of advantages such as high modularity, high efficiency, low switching frequency and low harmonics [1]–[3]. To ensure safety of power electronics devices and to minimize the power outage zone during line faults, MMC-HVDC grids are suggested to operate with high-speed DC circuit breakers at terminals of each transmission line, in order to isolate the faulted line as soon as possible (with typical data window of several milliseconds after the occurrence of the fault) [4]–[6]. After the isolation of the fault, in order to minimize the effort of searching for the fault in the faulted line and to reduce the power outage time as well as the operating costs, the location of the fault should be accurately determined with the short data window during faults [7]–[9]. Existing transmission line fault location methods in MMC-HVDC grids can be mainly classified into time domain and frequency domain methods.

Time domain methods can be further categorized into measurement based and model based methods in time domain. The most widely adopted measurement based methods in time domain are the traveling wave methods [10]. Single ended travelling wave fault location algorithms estimate the fault location using the arrival time difference of subsequent traveling waves at the local terminal of the line [11], [12]. The validity of single ended methods is based on accurate identification of the traveling wave reflected by the fault. Dual ended travelling wave methods utilize measurements at both ends of the line. A reliable communication channel between terminals of the line is typically required. They determine the location of the fault by the difference of the arrival time of the first waveform at both terminals [13]. Dual ended travelling wave methods can be categorized into methods that use synchronized measurements [14] or unsynchronized measurements [15]. Traveling wave methods encounter the following challenges. First, the methods require reliable detection of specific travelling waves (generated or reflected by the fault, etc.), which could be challenging especially during high resistance faults. Second, a very high sampling rate (hundreds of kilo samples per second or even higher) is typically required to ensure accurate fault location results.

To take full advantage of the transmission line models, researchers also propose model based fault location approaches in time domain. The main idea of these methods is to build the
relationship among the fault location, the available measurements and the transmission line model with fault. In [16], the 1-mode voltage distributions through the line from two terminals are estimated using voltage and current measurements at both terminals based on the Bergeron transmission line model. The location of the fault is obtained by intersecting the two voltage distribution curves. The method depends on the 1-mode Bergeron model of the transmission line, with lumped resistances and complete decoupling of the two-pole HVDC transmission line. Literature [17] combines the traveling wave theory with the Bergeron time domain method to locate faults using unsynchronized measurements. The method requires reliable detection of traveling wave head and is validated with a very high sampling rate of 1 MHz. A simplified R-L line model based fault location method is proposed in [18]. The high frequency components of the terminal measurements are filtered out and only low frequency measurements are utilized for fault location. The method utilizes the lumped R-L line model and the shunt capacitances are completely neglected.

Besides time domain methods, frequency domain methods are also studied in literatures which adopt the fault location information in the frequency spectrum of terminal measurements. The natural frequency based fault location methods for HVDC transmission lines are proposed in [19], [20]. These methods first derive the analytical relationship between the dominant frequency and the location of the fault. Afterwards, a high resolution spectra estimation tool is utilized to extract the natural frequencies and the fault location is obtained. The methods only require single ended measurements. Nevertheless, when the fault occurs near the local terminal of the line, the dominant natural frequency could be very large and exceed the maximum frequency of the spectrum. In this case, the fault location may not be accurately captured.

Dynamic state estimation (DSE) could be a promising way to track dynamics and estimate unknown variables [21], [22], and could be applied to solve fault location problems in DC transmission lines. In fact, in our previous work [23], [24], time domain model based fault location approaches using DSE have been studied and validated in AC transmission lines. Nevertheless, when applied to transmission lines in MMC-HVDC grids, the existing DSE based fault location methods encounter huge challenges. The characteristics of available measurements during faults in DC lines are very different from those in AC lines, such as short data window (several milliseconds), absence of fundamental frequency (50 Hz or 60 Hz) components, severe transients and rich high frequency components (due to low inertia of DC systems), etc. Consequently, the existing DSE based methods with nonlinear DSE procedure may not be applicable to transmission lines in MMC-HVDC grids due to large numerical error and extremely high computational burden.

To solve above issues, this paper proposes a novel transmission line fault location method in MMC-HVDC grids based on DSE and gradient descent. The method first accurately describes the physical laws that the DC transmission line with fault should obey through the linear high-fidelity dynamic model of the line (a set of matrix algebraic and differential equations), with any given fault location. The high-fidelity dynamic model is constructed by separating the entire transmission line into a large number of π equivalent two-pole line sections (hundreds of sections), which is a very good approximation of the fully distributed parameter model of the transmission line. Afterwards, the DSE procedure is applied to solve the dynamic states of the system and to check the consistency between the measurements and the dynamic model. Finally, the fault location is solved through gradient descent: the correct fault location result corresponds to the best consistency.

The contributions of the paper are summarized below:
- The proposed method utilizes a high-fidelity dynamic model of the transmission line, which accurately describe the physical laws of the DC lines in MMC-HVDC grids during faults;
- Compared to the existing DSE based methods which solve a highly nonlinear DSE problem, the proposed method only needs to solve a series of linear DSE problem; as a result, the proposed method is of much less numerical error and computational burden;
- The proposed method works with a short time window (5 ms) and a relatively low sampling rate (20k samples/sec).

The remainder of the paper is arranged as follows. Section II reviews the existing DSE based fault location method. Section III introduces the proposed fault location method based on DSE and gradient descent. Section IV demonstrates the numerical experiments, where the performance of the proposed method is compared to that of the existing DSE based method. Section V further discusses the effectiveness of the proposed method. Section VI concludes the paper.

II. REVIEW OF THE EXISTING DYNAMIC STATE ESTIMATION BASED TRANSMISSION LINE FAULT LOCATION METHOD

The main idea of the existing DSE based fault locating method for transmission line is to first build the nonlinear dynamic line model which considers the fault location as an extended state variable, and use DSE algorithm to solve the nonlinear dynamic model and find the fault location [23], [24]. Specifically for a two-pole DC transmission line, the nonlinear dynamic line model could be represented as a multi-section model, where each section is an n equivalent circuit, as shown in Fig. 1. The transmission line nonlinear dynamic model has the following format (here the control variable terms u(t) defined in [23] are not shown since no control variables are required for this fault location application),

\[
\begin{align*}
    z(t) &= Y_{eqx1}\dot{x}(t) + Y_{eqp1}p(t) + D_{eqx1}\dot{x}(t)/dt + C_{eqx1} \\
    0 &= Y_{eqx2}\dot{x}(t) + Y_{eqp2}p(t) + D_{eqx2}\dot{x}(t)/dt + C_{eqx2}
\end{align*}
\]
where the state vector \( x(t) \) includes section currents \( i_{l,k}^{(a)} \) and section voltages \( v_{k}^{(a)} \) and \( v_{k+1}^{(a)} \), \( k \) means section index (left side of fault: \( k = 1, \cdots, m \) and right side of fault: \( k = 1, \cdots, n \), \( a = l \) represents sections of left part and \( a = r \) represents sections of right part. The fault location \( l_f \) and fault resistance \( R_f \) can be included in the parameter vector \( p(t) \). Other matrices are constant coefficients. Detailed definitions can be found in Fig. 1 and [23]. It can be observed that the model is highly nonlinear.

To solve the states and parameters of the system, the parameters are treated as extended states of the system, and the differential equations of the dynamic line model in (1) are transformed into an algebraic form using quadratic integration,

\[
z(t, t_m) = h(xp(t, t_m))
\]

where \( t_m = t - \Delta t, \Delta t \) is the DSE time step, \( xp(t, t_m) = [x(t), \dot{x}(t)]^T = x, p \), \( xp(t, t_m) = [x(t, t_m), p(t, t_m)]^T \), and \( z(t, t_m) = [z(t), 0, 0, z(t, 0), 0]^T \).

Many DSE algorithms can be applied to solve (2). In [23], the unconstraint weighted least square method is utilized for each DSE time step at time \( t \). The solution is given with the following Newton’s iterative algorithm until convergence,

\[
xp(t, t_m)^{m+1} = xp(t, t_m)^m - (H^T WH)^{-1} \times H^T W (h(xp(t, t_m)^m) - z(t, t_m))
\]

where the weight matrix is \( W = diag[1/\sigma_1^2, 1/\sigma_2^2, \ldots] \), and \( \sigma_i \) is the \( i \)th measurement standard deviation in \( z(t, t_m) \), and the Jacobian matrix is \( H = \partial h(xp(t, t_m))/\partial xp(t, t_m) \).

III. PROPOSED FAULT LOCATION METHOD BASED ON DYNAMIC STATE ESTIMATION AND GRADIENT DESCENT

The existing DSE based fault location method treats the fault location \( l_f \) as an extended state of the dynamic line model. Since the variable \( l_f \) is strongly coupled with a large number of state variables (section currents \( i_{l,k}^{(a)} \) and section voltages \( v_{k}^{(a)} \) and \( v_{k+1}^{(a)} \), as defined in Fig. 1), the dynamic line model of the existing method is highly nonlinear. In fact, the effectiveness of the existing fault location method has been well validated in AC transmission lines. However, the existing method may encounter additional challenges when applied to DC transmission lines (especially for lines in MMC-HVDC grids). Faults in DC lines have the following characteristics compared to those in AC lines. First, the fundamental frequency of 50 or 60 Hz is absent in DC transmission lines. Second, the inertia of the power electronic interfaced systems is much lower compared to traditional AC systems. Third, the DC transmission line is usually isolated very fast after the occurrence of the fault: the time window for available voltage and current measurements during faults are extremely short (for example 5ms). As a result, the transients of the available measurements during faults in DC lines are much more intense compared to those in AC lines. Consequently, there are two main limitations of the existing method when applied to DC transmission lines.

1) Numerical error: To accurately track the severe transients of the voltages and currents during faults in DC lines, the DSE time step should be small enough. Correspondingly, to ensure the accuracy of the dynamic line model, the section number of the line model should be very large (hundreds of sections for DC lines instead of tens of sections in AC lines). In this case, the condition number of matrix \( H^T WH \) is large due to the small DSE time step and large section number. Therefore, a large numerical error could be generated when the inverse of the matrix is solved and updated in each Newton’s iteration and each DSE time step. Further, the large numerical error might cause divergence issues due to the high nonlinearity of the DSE problem.

2) High computational burden: Large section number of the dynamic line model results in a high-dimensional matrix \( H^T WH \). Since the Jacobian matrix \( H \) is a function of the extended state vector \( xp(t, t_m)^m \), the inverse of the high-dimensional matrix \( H^T WH \) should be updated in each Newton’s iteration and also in each DSE time step. This will result in extremely high computational burden for the existing DSE based fault location method. To overcome the aforementioned limitations, an improved DSE based fault location approach is proposed. The main idea is to avoid introducing the fault location \( l_f \) and fault resistance \( R_f \) as extended states of the transmission line dynamic model. As a result, the transmission line dynamic model can be simplified as a linear model and the solution procedure via DSE is also much simplified. The limitations such as numerical errors and computational burden can be much mitigated. Afterwards, the fault location can be obtained via gradient descent. Details of the proposed method are provided next.

A. Highly-Fidelity Linear Dynamic Line Model During Fault

The method first builds the high-fidelity linear dynamic line model during fault. The main idea is to separate the entire transmission line into a large number of \( \pi \) equivalent sections. The number of sections could be very large (hundreds of sections) to ensure that the model is a very close approximation of the fully distributed parameter transmission line in MMC-HVDC grids. In addition, the fault location \( l_f \) and resistance \( R_f \) are given before the DSE procedure to ensure linearity of model. Note that the given fault location and resistance values are calculated from the last gradient descent step or the initial value (if it is the first step). The linear dynamic model is developed as,

\[
0 = Y_{eqx1} x(t) + D_{eqx1} dx(t)/dt
\]

\[
0 = Y_{eqx2} x(t) + D_{eqx2} dx(t)/dt
\]
where the state vector $x(t) = [v_1(i(t)), v_2(i(t)), \ldots, v_{n+1}(i(t)), v_{n+1}(r(t)), \ldots, v_{n+1}(r(t)), i_{L1}(i(t)), \ldots, i_{Ln}(i(t)), i_{L1}(r(t)), \ldots, i_{Ln}(r(t))]^T$, the actual measurement vector $z(t) = [v_1(i(t)), v_{n+1}(r(t)), i_{L1}(i(t)), i_2(r(t))]^T$, the matrices $Y_{eq1}$, $Y_{eq2}$, $D_{eq1}$, $D_{eq2}$, and the state vector $W$ are shown at the bottom of this page, and $I_j$ is the identity matrix with the dimension of $j$. The matrices $Y_{eq1}$, $Y_{eq2}$, $D_{eq1}$, $D_{eq2}$ are block diagonal matrices with $n$-1 $G_r$ matrices along the diagonal. $E_j = [0_{2\times(2m)}, M_{fault} - (G_1 + G_2)/2]$, $Y_{eq1} = [I_2, -I_2]$, $Y_{eq2} = [I_2, I_2]$, $Y_{eq3} = [I_2, I_2]$, $Y_{eq4}$ are block diagonal matrices with $n$-1 $G_r$ matrices, $m R_l$ matrices and $n R_r$ matrices along the diagonal, respectively. $R_l = R_l \cdot l/m$, $L_l = L_l \cdot l/m$, $G_l = G_l \cdot l/m$, $C_l = C_l \cdot l/m$, $R_r = R_r \cdot (l-l_j)/n$, $L_r = L_r \cdot (l-l_j)/n$, $G_r = G_r \cdot (l-l_j)/n$, $C_r = C_l \cdot (l-l_j)/n$, $R_l, L_l, G_l$ and $C_l$ are series resistance, series reactance, shunt conductance and shunt capacitance matrices along the diagonal, respectively.

$R_l = R_l \cdot l/m$, $L_l = L_l \cdot l/m$, $G_l = G_l \cdot l/m$, $C_l = C_l \cdot l/m$, $R_r = R_r \cdot (l-l_j)/n$, $L_r = L_r \cdot (l-l_j)/n$, $G_r = G_r \cdot (l-l_j)/n$, $C_r = C_l \cdot (l-l_j)/n$, $R_l, L_l, G_l$ and $C_l$ are series resistance, series reactance, shunt conductance and shunt capacitance matrices along the diagonal, respectively.

The algebraic form of the linear dynamic model is,

$$z(t, t_m) = Y_{eq1} x(t, t_m) - B_{eq}$$

where $t_m = t - \Delta t$, $\Delta t$ is the DSE time step, $x(t, t_m) = [x(t), x(t_m)]^T$, $z(t, t_m) = [z(t), 0, z(t_m), 0]^T$, $B_{eq} = \ldots$
\[-N_{eqx} \cdot \hat{x}(t - 2\Delta t) - M_{eq} \cdot \hat{z}(t - 2\Delta t), \text{ and}\]
\[Y_{eqx} = \begin{bmatrix} Y_{eqx1} + 2D_{eqx1}/\Delta t & -4D_{eqx1}/\Delta t \\
Y_{eqx2} + 2D_{eqx2}/\Delta t & -4D_{eqx2}/\Delta t \\
D_{eqx1}/(4\Delta t) & Y_{eqx1} + D_{eqx1}/(4\Delta t) \\
D_{eqx2}/(4\Delta t) & Y_{eqx2} + D_{eqx2}/(4\Delta t) \end{bmatrix}, \]
\[N_{eqx} = \begin{bmatrix} -Y_{eqx1} + 2D_{eqx1}/\Delta t \\
-Y_{eqx2} + 2D_{eqx2}/\Delta t \\
Y_{eqx1}/2 - 5D_{eqx1}/(4\Delta t) \\
Y_{eqx2}/2 - 5D_{eqx2}/(4\Delta t) \end{bmatrix}, \]
\[M_{eq} = [I_2 \ 0_{(4m+4n-2)\times2} \ -0.5I_2 \ 0_{(4m+4n-2)\times2}]^T. \]

### B. Dynamic State Estimation Procedure

Here the unconstrained weighted least square method is selected as an example,
\[
\min_{x(t,t_m)} J(t) = (r(t,t_m))^TW(r(t,t_m)) \]  \(6\)
where the residual is defined as the difference between the estimated measurements and actual measurements,
\[r(t,t_m) = Y_{eqx} \hat{x}(t,t_m) - B_{eq} - \hat{z}(t,t_m) \]  \(7\)
Since the dynamic model is linear, the solution does not require iterations. The best estimated state vector \(\hat{x}(t,t_m)\) for each DSE time step at time \(t\) is,
\[\hat{x}(t,t_m) = (Y_{eqx}^TWY_{eqx})^{-1}Y_{eqx}^TW(z(t,t_m) + B_{eq}) \]  \(8\)
where the weight matrix is \(W = \text{diag}\{1/\sigma_1^2, 1/\sigma_2^2, \ldots\}\), and \(\sigma_i\) is the \(i\)th measurement standard deviation in \(z(t,t_m)\).

Note that here the matrix that needs to be inversed is \(Y_{eqx}^TWY_{eqx}\). With given fault location and fault resistance, the matrix \(Y_{eqx}^TWY_{eqx}\) is constant, independent of state vector \(x(t,t_m)\). Therefore, compared to the existing method which needs to calculate the inverse matrix in each Newton’s iteration and each DSE time step, the proposed method can solve the inverse of the matrix \(Y_{eqx}^TWY_{eqx}\) before the DSE procedure and utilize the same inverse matrix through all the DSE time steps. This will largely mitigate the issues such as numerical errors, convergence, and computational burden.

### C. Fault Location via Gradient Descent of Chi-Square Value

It can be observed from part III.A and B that the execution of the DSE is with the given fault location \(l_f\) and resistance \(R_f\). If the measurements are inconsistent with the linear dynamic model of the line with fault, we blame this on the inaccuracy of the linear dynamic model, i.e. the given fault location \(l_f\) and resistance \(R_f\) are not accurate. On the other hand, if the measurements are consistent with the linear dynamic model, the given fault location \(l_f\) and resistance \(R_f\) are trustworthy. Therefore, the fault location can be obtained by checking the consistency between the measurements and model.

In fact, the consistency at time \(t\) can be represented by the chi-square value \(\chi(t)\) (the weighted sum of residual squares) during the DSE procedure: \(\chi(t)\) can be calculated by substituting the best estimated state vector \(\hat{x}(t,t_m)\) into \(6\). The generalized consistency can be quantified by taking the average of the chi-square value during a user-defined time window of the DSE procedure.

To sum up, the average chi-square value \(\hat{y}\) indicates the consistency between the measurements and the linear dynamic model of the line with fault, and is a function of fault location \(l_f\) and fault resistance \(R_f\). In addition, actual fault location and fault resistance will result in minimum average chi-square value (best consistency). Therefore, the fault location can be obtained by solving the following optimization problem,
\[\min_{l_f,R_f} y = \chi(l_f,R_f) \]  \(9\)
where \(\chi(\cdot)\) expresses \(y\) as function of \(l_f\) and \(R_f\).

To solve the optimization problem, here the gradient descent method is adopted as an example. The iterative procedure is,
\[(l_f^{(v+1)}, R_f^{(v+1)}) = [l_f^{(v)}, R_f^{(v)}] - \alpha^{(v)} \nabla \chi(l_f^{(v)}, R_f^{(v)}) \]  \(10\)
where \(\alpha^{(v)}\) is the step size that satisfies the Armijo condition [25] and \(\nabla \chi(l_f^{(v)}, R_f^{(v)})\) is the gradient that could be numerically calculated through \(11\),
\[\nabla \chi(l_f^{(v)}, R_f^{(v)}) = \begin{bmatrix} (\chi(l_f^{(v)}, R_f^{(v)}) + \Delta l_f, R_f^{(v)}) - \chi(l_f^{(v)}, R_f^{(v)})/\Delta l_f \\
(\chi(l_f^{(v)}, R_f^{(v)}) + \Delta R_f, R_f^{(v)}) - \chi(l_f^{(v)}, R_f^{(v)})/\Delta R_f \end{bmatrix}^T \]  \(11\)
where \(\Delta l_f\) and \(\Delta R_f\) are small perturbations.

The flow chart comparison between the existing method and the proposed method is shown in Fig. 2. It can be observed that, the proposed method solves linear DSE problems with constant inverse matrices, while the existing method solves highly nonlinear DSE problems with inverse matrix that requires to be updated in each Newton’s iteration and each DSE time step. This advantage enables more accurate, fast and reliable fault location estimation of the proposed method compared to that of the existing method for transmission lines in MMC-HVDC grids.

In fact, the gradient descent method converges to a local minimum with given initialization [25]. Since there might be several local minima of the function \(\chi(l_f, R_f)\), the gradient descent processes with random initial values of \(l_f\) and \(R_f\) are executed in parallel to ensure convergence to the global minimum.

### IV. Numerical Experiments

An example \(\pm 3200 \text{ kV}\) bipolar MMC-HVDC grid is shown in Fig. 3. The transmission line of interest is the \(200 \text{ km}\) transmission line S-R. The parameters of the MMC-HVDC grid and parameter matrices of the transmission line of interest are shown in Table III and Table IV, respectively. The synchronized voltage and current instantaneous measurements are installed at both terminals of the line, with \(20 \text{ kило samples/sec}\) sampling rate. The voltage and current transducers are not modeled in this study. Faults with different types, locations and resistances are simulated in PSCAD/EMTDC. The simulation time step of the
MMC-HVDC system in PSCAD/EMTDC is set as 1 μs. During simulation, the fully distributed parameter line is assumed to be frequency independent and is simulated using a multi-section model with extensive number of π sections (here 1000 π sections is adopted). Note that this simulation method presents a closer approximation to the fully distributed parameter transmission line compared to the Bergeron model since the Bergeron model in PSCAD only considers lumped resistances. The available data window is 5 ms after the occurrence of the fault. To ensure that the traveling wave generated by the fault has reached at least one terminal of the line, the first 0.33 ms of the data is not utilized. Here the DSE time step is selected as 10 μs to accurately track the system dynamics during the severe transients. The cubic spline interpolation is utilized to complete the measurement set.

Several settings of the proposed method are as follows. The section numbers of the transmission line model are selected as \(m = 200\), \(n = 200\), which is a very close approximation of a fully distributed parameter transmission line in MMC-HVDC grids. The average chi-square value during last 1 ms is used. The gradient descent algorithm is initialized with 10 points that are randomly selected from \(0 < l_f^0 < 200 \times 10^3 \text{m}\) and \(0 < R_f^0 < 500 \text{ohm}\). The perturbations to numerically calculate the gradient are set as \(\Delta l_f = 1 \text{m}\) and \(\Delta R_f = 0.1 \text{ohm}\). The coverage condition of the gradient descent is \(\| \nabla \chi(l_f^0, R_f^0) \|^2 \leq \varepsilon\), where \(\varepsilon = 10^{-4}\) per unit. The performance of the proposed method is compared to the existing method in [23] via the following test cases. Here the section numbers of the transmission line model of the existing method are also selected as \(m = 200\),

**TABLE III**

<table>
<thead>
<tr>
<th>Parameters of the MMC-HVDC Grid (MMC 1 and MMC 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
</tr>
<tr>
<td>---------------------------</td>
</tr>
<tr>
<td>AC rated voltage (kV)</td>
</tr>
<tr>
<td>DC rated voltage (kV)</td>
</tr>
<tr>
<td>DC line reactor (H)</td>
</tr>
<tr>
<td>Arm reactor (H)</td>
</tr>
<tr>
<td>Submodules capacitor (μF)</td>
</tr>
</tbody>
</table>

**TABLE IV**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Matrices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Series resistance (per meter)</td>
<td>(\begin{bmatrix} 1.0955 &amp; 0 \ 0 &amp; 1.0955 \end{bmatrix} \times 10^{-4} \text{ohm/m} )</td>
</tr>
<tr>
<td>Series inductance (per meter)</td>
<td>(\begin{bmatrix} 1.4633 &amp; 0.6561 \ 0.6561 &amp; 1.4633 \end{bmatrix} \times 10^{-6} \text{H/m} )</td>
</tr>
<tr>
<td>Shunt capacitance (per meter)</td>
<td>(\begin{bmatrix} 6.3154 &amp; -0.8407 \ -0.8407 &amp; 6.3154 \end{bmatrix} \times 10^{-12} \text{F/m} )</td>
</tr>
<tr>
<td>Shunt conductance (per meter)</td>
<td>(\theta_{\text{sh}}) mho/m</td>
</tr>
</tbody>
</table>
Fig. 4. Results of the existing method for the 0.01 ohm positive pole to ground fault at 50 km from side S. (a) Condition number of $H^TWH$ (b) Fault location results.

$n = 200$ to make them comparable. For both the existing and the proposed method, the absolute fault location error in percentage is defined as,

$$\text{Absolute Error} = \left| \frac{\text{Estimated Location} - \text{Actual Location}}{\text{Total Length of the Line}} \right| \times 100\% \quad (12)$$

A. Test Case 1: Single Pole to Ground Faults

A positive pole to ground fault with 0.01 ohm fault resistance occurs at 50 km from side S and at time $t = 0.5$ s. The results of the existing method are depicted in Fig. 4. Fig. 4(a) shows the condition number of the matrix $H^TWH$ at the end of each DSE time step. It can be observed that the condition numbers reach the order of $10^{32}$, which means that the calculation of the inverse matrix will probably generate large numerical errors. The fault location results of the existing method are shown in Fig. 4(b). It can be observed that the fault location results are obviously not correct since the estimated fault location is much more than the entire length of the transmission line of interest.

The results of the proposed method are demonstrated next. Fig. 5 depicts the functional relationship of $y = \chi(l_f,R_f)$, i.e., the average chi-square value with different fault location and fault resistances. The global minimum value of $y$ is achieved near $l_f = 50$ km and $R_f = 0$ ohm, which matches the actual fault location and fault resistance. The proposed method finds the minimum value at $l_f = 49.42$ km and $R_f = 0.0023$ ohm. The absolute fault location error is 0.29%. Note that this figure is only for demonstration purpose (same with Fig. 8, 11 and 14), to show that the proposed method can obtain the global optimal solution, which corresponds to minimum value of $y$ and best consistency.

The proposed method is further validated via a group of positive pole to ground faults with different fault resistances (0.01, 1, 5 and 10 ohm) and fault locations (every 10 km from side S to side R). The fault location results are summarized in Fig. 6 and Table V. It can be observed that the average absolute errors are less than 0.19% and the maximum absolute errors are less than 0.46%. Therefore, the proposed method is able to accurately locate faults in this test case.

B. Test Case 2: Pole to Pole Faults

A pole to pole fault with 0.01 ohm fault resistance occurs at 50 km from side S and at time $t = 0.5$ s. The results of the existing method are depicted in Fig. 7. Fig. 7(a) shows that the condition numbers of the matrix $H^TWH$ reach the order of $10^{46}$ at time $2.2$ ms after the occurrence of the fault. The fault location results of the existing method in Fig. 7(b) reach unrealistic values.

The results of the proposed method are demonstrated next. Fig. 8 depicts the functional relationship of $y = \chi(l_f,R_f)$. The global minimum value of $y$ is achieved near $l_f = 50$ km and $R_f = 0$ ohm, which matches the actual fault location and fault resistance. The proposed method finds the minimum value at $l_f = 49.42$ km and $R_f = 0.0023$ ohm. The absolute fault location error is 0.29%.
Fig. 8. Results of the proposed method for the 0.01 ohm pole to pole fault at 50 km from side S.

Fig. 9. Fault location results for a group of pole to pole faults (low fault resistances and different locations).

<table>
<thead>
<tr>
<th>Fault resistance (Ω)</th>
<th>Average absolute error (%)</th>
<th>Max absolute error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.1420</td>
<td>0.5507</td>
</tr>
<tr>
<td>1</td>
<td>0.1991</td>
<td>0.5446</td>
</tr>
<tr>
<td>5</td>
<td>0.1889</td>
<td>0.5728</td>
</tr>
<tr>
<td>10</td>
<td>0.1985</td>
<td>0.5345</td>
</tr>
</tbody>
</table>

= 50.481 km and \( R_f = 0.8803 \) ohm. The absolute fault location error is 0.241%.

The proposed method is further validated via a group of pole to pole faults with different fault resistances (0.01, 1, 5 and 10 ohm) and different fault locations (every 10 km from side S to side R). The fault location results are summarized in Fig. 9 and Table VI. It can be observed that the average absolute errors are less than 0.2% and the maximum absolute errors are less than 0.58%. Therefore, the proposed method is able to accurately locate faults in this test case.

C. Test Case 3: Double Pole to Ground Faults

A double pole to ground fault with 0.01 ohm fault resistance occurs at 50 km from side S and at time \( t = 0.5 \) s. The results of the existing method are depicted in Fig. 10. Fig. 10(a) shows the condition numbers of the matrix \( H^T W H \) reach the order of \( 10^{18} \). The fault location results of the existing method in Fig. 10(b) oscillate and also fail to reach accurate values.

Fig. 10. Results of the existing method for the 0.01 ohm double pole to ground fault at 50 km from side S. (a) Condition number of \( H^T W H \) (b) Fault location results.

Fig. 11. Results of the proposed method for the 0.01 ohm double pole to ground fault at 50 km from side S.

The results of the proposed method are demonstrated next. Fig. 11 depicts the functional relationship of \( y = \chi(l_f, R_f) \). The global minimum value of \( y \) is achieved near \( l_f = 50 \) km and \( R_f = 0 \) ohm, which matches the actual fault location and fault resistance. The proposed method finds the minimum value at \( l_f = 50.507 \) km and \( R_f = 0.5221 \) ohm. The absolute fault location error is 0.254%.

The proposed method is further validated via a group of double pole to ground faults with different fault resistances (0.01, 1, 5 and 10 ohm) and different fault locations (every 10 km from side S to side R). The fault location results are summarized in Fig. 12 and Table VII. It can be observed that the average absolute errors are less than 0.19% and the maximum absolute errors are less than 0.55%. Therefore, the proposed method is able to accurately locate faults in this test case.
TABLE VII  
AVERAGE AND MAXIMUM ABSOLUTE ERRORS FOR A GROUP OF DOUBLE POLE TO GROUND FAULTS (LOW FAULT RESISTANCES AND DIFFERENT LOCATIONS)

<table>
<thead>
<tr>
<th>Fault resistance (Ω)</th>
<th>Average absolute error (%)</th>
<th>Max absolute error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.1335</td>
<td>0.5497</td>
</tr>
<tr>
<td>1</td>
<td>0.1804</td>
<td>0.5170</td>
</tr>
<tr>
<td>5</td>
<td>0.1074</td>
<td>0.3022</td>
</tr>
<tr>
<td>10</td>
<td>0.1234</td>
<td>0.4386</td>
</tr>
</tbody>
</table>

Fig. 13. Results of the existing method for the 200 ohm positive pole to ground fault at 50 km from side S. (a) Condition number of $H^TWH$ (b) Fault location results.

D. Test Case 4: High Resistance Faults

A positive pole to ground fault with 200 ohm fault resistance occurs at 50 km from side S and at time $t = 0.5$ s. The results of the existing method are depicted in Fig. 13. Fig. 13(a) shows that the condition numbers of the matrix $H^TWH$ reach the order of $10^{21}$ during the fault. The fault location results of the existing method in Fig. 13(b) also reach unrealistic values.

The results of the proposed method are demonstrated next. Fig. 14 depicts the functional relationship of $y = \chi(l_f, R_f)$. The global minimum value of $y$ is achieved near $l_f = 50$ km and $R_f = 200$ ohm, which matches the actual fault location and fault resistance. The proposed method finds the minimum value at $l_f = 49.36$ km and $R_f = 199.6725$ ohm. The absolute fault location error is 0.32%.

The proposed method is further validated via a group of positive pole to ground faults with different fault resistances (200, 300, 400 and 500 ohm, to cover extreme cases) and different fault locations (every 10 km from side S to side R). The fault location results are summarized in Fig. 15 and Table VIII. It can be observed that the average absolute errors are less than 0.43% and the maximum absolute errors are less than 1.29%. Therefore, the proposed method presents adequate accuracy during high impedance faults.

TABLE VIII  
AVERAGE AND MAXIMUM ABSOLUTE ERRORS FOR A GROUP OF POSITIVE POLE TO GROUND FAULTS (HIGH FAULT RESISTANCES AND DIFFERENT LOCATIONS)

<table>
<thead>
<tr>
<th>Fault resistance (Ω)</th>
<th>Average absolute error (%)</th>
<th>Max absolute error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.2469</td>
<td>0.6966</td>
</tr>
<tr>
<td>300</td>
<td>0.2782</td>
<td>0.8249</td>
</tr>
<tr>
<td>400</td>
<td>0.3084</td>
<td>1.0679</td>
</tr>
<tr>
<td>500</td>
<td>0.4211</td>
<td>1.2898</td>
</tr>
</tbody>
</table>

Fig. 15. Fault location results for a group of positive pole to ground faults (high fault resistances and different locations).

V. DISCUSSION

To further ensure the effectiveness of the proposed fault location method, the effects of different measurement errors, parameter errors and section numbers are discussed next. The example test system as well as the settings of the proposed method are the same as those in part IV. For each subsection, 0.01 ohm positive pole to ground faults are studied as examples.

A. Effect of Measurement Errors

The Gaussian distributed errors with 0.2%, 0.5% and 1% standard deviations are added to the instantaneous measurements, respectively. The fault location results are summarized in Fig. 16 and Table IX. It can be observed that the average absolute errors are less than 0.21% and the maximum absolute errors are less than 0.44%.
B. Effect of Parameter Errors

The 0.2%, 0.5% and 1% parameter errors are added to all parameter matrices in the proposed method, respectively. The fault location results are summarized in Fig. 17 and Table X. It can be observed that the average absolute errors are less than 0.31% and the maximum absolute errors are less than 1.3%. One can observe that the fault location errors of the proposed method increase with larger parameter errors. In practice, parameter identification approaches can be applied to minimize the parameter errors of the transmission line of interest.

C. Effect of Section Numbers

The section numbers of the linear dynamic model of the line with fault in the proposed method are selected as $m = n = 10$, 20, 50, 100, 200 and 400, respectively. The fault location results are summarized in Fig. 18 and Table XI. It can be observed that absolute fault location errors are generally smaller with larger section numbers. However, the fault location accuracy remains similar if the section number is larger than 200. Therefore, the section number selection of $m = n = 200$ is adequate to ensure fault location accuracy. Note that the optimum section numbers $m$ and $n$ may increase for longer transmission lines.

D. Effect of Data Window Sizes

In fact, the previous 5 ms time window selection is consistent with the requirements of practical MMC-HVDC grids: for example in Zhangbei MMC-HVDC grid, the protection system needs to detect and isolate faults within 6 ms (including 3 ms for the protection system to detect the fault, and 3 ms for the DC circuit breaker to isolate the fault) [6]. Nevertheless, in practice especially for severe faults, there are still possibilities that the faults are isolated even faster than 5 ms. Next, the time windows of 3, 4 and 5 ms are adopted respectively, and the fault location results are summarized in Fig. 19 and Table XII. It can be observed that the fault location errors slightly increase with shorter time window. Nevertheless, the average absolute errors

---

### Table IX

<table>
<thead>
<tr>
<th>Measurement error (%)</th>
<th>Average absolute error (%)</th>
<th>Max absolute error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.1855</td>
<td>0.3887</td>
</tr>
<tr>
<td>0.5</td>
<td>0.1907</td>
<td>0.4043</td>
</tr>
<tr>
<td>1</td>
<td>0.2022</td>
<td>0.4323</td>
</tr>
</tbody>
</table>

### Table X

<table>
<thead>
<tr>
<th>Parameter error (%)</th>
<th>Average absolute error (%)</th>
<th>Max absolute error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.2067</td>
<td>0.4293</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2430</td>
<td>0.7330</td>
</tr>
<tr>
<td>1</td>
<td>0.3038</td>
<td>1.2869</td>
</tr>
</tbody>
</table>

### Table XI

<table>
<thead>
<tr>
<th>Section number</th>
<th>Average absolute error (%)</th>
<th>Max absolute error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.7369</td>
<td>3.1826</td>
</tr>
<tr>
<td>20</td>
<td>0.5828</td>
<td>2.5971</td>
</tr>
<tr>
<td>50</td>
<td>0.3743</td>
<td>1.3571</td>
</tr>
<tr>
<td>100</td>
<td>0.2135</td>
<td>0.5474</td>
</tr>
<tr>
<td>200</td>
<td>0.1827</td>
<td>0.3738</td>
</tr>
<tr>
<td>400</td>
<td>0.1854</td>
<td>0.3347</td>
</tr>
</tbody>
</table>

### Table XII

<table>
<thead>
<tr>
<th>Window size (ms)</th>
<th>Average absolute error (%)</th>
<th>Max absolute error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.1827</td>
<td>0.3738</td>
</tr>
<tr>
<td>4</td>
<td>0.1837</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>0.2726</td>
<td>0.5</td>
</tr>
</tbody>
</table>
are less than 0.2726% and the maximum absolute errors are less than 0.5% for all scenarios.

E. Effect of Sampling Rates

Here the sampling rates of 100, 50 and 20 kilo samples/sec are adopted respectively. The DSE time step remain unchanged as 10μs and the cubic spline interpolation is applied to complete the measurement set if necessary. The fault location results are summarized in Fig. 20 and Table XIII. It can be observed that the fault location errors decrease slightly with sampling rates higher than 20 kilo samples/sec. The average absolute errors are less than 0.19% and the maximum absolute errors are less than 0.38%.

VI. CONCLUSION

In this paper, a new transmission line fault location scheme in MMC-HVDC grids based on dynamic state estimation (DSE) and gradient descent is proposed. Existing DSE based fault location methods solve highly nonlinear DSE problems to determine the fault location. As a result, when applied to transmission lines in MMC-HVDC grids, the validity of the existing method is limited due to large numerical errors and high computational burden. To overcome these limitations, the proposed method first establishes a linear dynamic model of the DC transmission line during the fault. Afterwards, the consistency between the measurements and the linear dynamic model is obtained by solving the linear DSE problem. Finally, the fault location is accurately determined through gradient descent by observing the fact that the actual fault location must correspond to the best consistency. Extensive numerical experiments show that the proposed method works with a relatively low sampling rate of 20 kilo samples per second and a short data window of 5 ms, and can accurately locate faults regardless of the fault types, resistances and locations. Note that the proposed method can be similarly extended to cables or non-homogeneous transmission circuits.

The effectiveness of the proposed fault location methodology on transmission circuits with other complex structures or extensive lengths will be studied in future publications. In addition, the proposed method does not consider frequency dependent parameters of the transmission lines, which may generate fault location errors especially during severe transients. This issue will also be studied in future publications.

REFERENCES

[1] A. Nami, J. Liang, F. Dijkhuizen, and G. D. Demetriades, “Modular multi- 
tile converters for HVDC applications: Review on converter cells and 
Jan. 2015.
[2] Y. Li, G. Tang, J. Ge, and Z. He, “Modeling and damping control of 
“Detailed and averaged models for a 401-Level MMC–HVDC system,” 
HVDC circuit breaker sizing,” IEEE Trans. Power Del., vol. 35, no. 1, 
[5] N. Tong et al., “Local measurement-based ultra-high-speed main protec-
tion for long distance VSC-MTDC,” IEEE Trans. Power Del., vol. 34, 
traveling wave protection method with adaptive threshold value and its 
application in HVDC grids,” IEEE Trans. Power Del., vol. 35, no. 4, 
wave-based line fault location in star-connected multiterminal HVDC 
systems,” IEEE Trans. Power Del., vol. 27, no. 4, pp. 2286–2294, 
on pulse injection from hybrid HVDC breaker,” IEEE Trans. Power Del., 
mision line fault location method with full consideration of distributed 
parameters and line asymmetry,” IEEE Trans. Power Del, to be published, 
doi: 10.1109/TPWDR.2020.2974294.
[10] D. Tzelepis, G. Fusiek, A. Dysko, P. Niewczas, C. Booth, and X. Dong, 
“Novel fault location in MTDC grids with non-homogeneous transmission 
lines utilizing distributed current sensing technology,” IEEE Trans. Smart, 
evaluation of single-end fault locator for two-terminal HVTD transmission 
fault location method in DC transmission line based on wave front infor-
references to HVDC fault location,” IEEE Trans. Power Del, vol. 8, no. 3, 
of DC line faults in conventional HVDC systems with segments of cables 
and overhead lines using terminal measurements,” IEEE Trans. Power Del., 
transmission lines,” IEEE Trans. Power Del., vol. 25, no. 2, pp. 1203–1209, 
Apr. 2010.
HVDC transmission lines under unsynchronized two-end measurement 
and uncertain line parameters,” IEEE Trans. Power Del., vol. 30, no. 3, 


Binglin Wang (Student Member, IEEE) received the B.S. degree in electrical engineering and intelligent control from the Xi’an University of Technology, Xi’an, Shaanxi, China, in 2018. He is currently working towards the Ph.D. degree in electrical engineering, with the School of Information Science and Technology, ShanghaiTech University, Shanghai, China. His research interests include protection, fault location and state estimation of HVAC and HVDC transmission lines.

Yu Liu (Member, IEEE) received the B.S. and M.S. degrees in electrical power engineering from Shanghai Jiao Tong University, Shanghai, China, in 2011 and 2013, respectively, and the Ph.D. degree in electrical and computer engineering from the Georgia Institute of Technology, Atlanta, GA, USA, in 2017. He is currently a Tenure-Track Assistant Professor with the School of Information Science and Technology, ShanghaiTech University, Shanghai, China. His research interests include modeling, protection, fault location, and state/parameter estimation of power systems and power electronic systems.

Rui Fan (Member, IEEE) received the B.S. degree from the Huazhong University of Science and Technology, Wuhan, Hubei, China, in 2011, and the M.S. and Ph.D. degrees from the Georgia Institute of Technology, Atlanta, GA, USA, in 2012 and 2016, respectively, all in electrical engineering. He is currently an Assistant Professor with the Department of Electrical and Computer Engineering at the University of Denver. His research interests include smart cities, power system protection and control, resilience and stability, and data-driven analysis.

Dayou Lu (Student Member, IEEE) received the B.S. degree in electrical engineering and automation from the Hubei University of Science and Technology, Wuhan, Hubei, China, in 2017. He is currently working towards the Ph.D. degree in electrical engineering, with the School of Information Science and Technology, ShanghaiTech University, Shanghai, China. His research interests include modeling, protection and fault location of transmission lines.

Kang Yue (Student Member, IEEE) received the B.S. degree in electrical engineering and automation from the Hefei University of Technology, Hefei, China, in 2017. She is currently working towards the Ph.D. degree in electrical engineering, with the School of Information Science and Technology, ShanghaiTech University, Shanghai, China. Her research interests include fault diagnosis, state estimation and parameter identification of power electronic systems.