An Improved Current Differential Protection Scheme on Non-Homogeneous Transmission Lines Considering Fully Distributed Parameter Model and Line Asymmetry

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Abstract—Current differential protection is widely adopted in the high voltage AC transmission line with high sensitivity and selectivity. The capacitive charging current in long distance non-homogeneous transmission lines with overhead lines and underground cables may be too large to be ignored. Therefore, accurate charging current compensation must be carefully considered. In this paper, an improved current differential protection is proposed for non-homogeneous transmission lines. The method can fully compensate the capacitive charging current by fully considering the distributed parameters and at the same time the line asymmetry. Numerical experimental results demonstrate that the proposed protection method can accurately compensate the charging current. In addition, compared to the legacy current differential protection methods, the proposed method presents improved dependability during internal faults, and improved security during external faults and other system transient.

Keywords—non-homogeneous transmission lines, current differential protection, capacitive charging current compensation

I. INTRODUCTION

Faults may sometimes occur on transmission lines. Reliable transmission line protection is essential to isolate the faulted line immediately after the occurrence of the fault, and therefore ensure safety of power equipment, personnel and the overall power system [1-3]. In recent years, non-homogeneous (also known as mixed, hybrid) transmission lines are widely adopted in practical power systems, which are good solutions for connecting the remote off-shore wind power to the existing grids. Due to the complex topology of the non-homogeneous transmission systems, protection of the non-homogeneous lines could be more challenging compared to that of the classic transmission lines. Legacy transmission line protection schemes mainly include single-ended methods and dual-ended methods, with details shown next.

Single-ended line protection schemes utilize measurements at the local terminal of the line and does not require communications from the remote ends, including overcurrent protection [4], directional overcurrent protection [5], and distance protection [6-7]. However, the main limitation of single-ended methods is that they cannot distinguish between internal faults and external faults at the remote end of the line (since the boundary between the internal/external faults is not obvious). Therefore in practice, the single-ended protective relays are usually designed with zone 1 (fast response, covering 75%-85% of the line) and zone 2 (intentional delays, covering the entire length of the line and 15%-25% of the next line), and the coordination among relays should be carefully considered [8]. Dual-ended line protection schemes (also known as pilot protection schemes) utilize the measurements at both terminals of the transmission line.

Proper communication channels are typically required. With more information from the remote end, the dual-ended line protection usually presents improved selectivity and reliability compared to the single-ended methods. One common pilot protection is directional comparison scheme: directional overcurrent relays or distance relays can be applied in the protection scheme [1]. Directional comparison scheme can be further classified into direct underreaching transfer trip (DUTT), permissive underreaching transfer trip (PUTT) and permissive overreaching transfer trip (POTT) [9-10]. However, these methods are affected by the large shunt admittances of the line under protection, and may encounter sensitivity issues during high impedance fault.

Another common pilot protection scheme is the current differential protection [9-12], which is to check the Kirchoff’s Current Law (KCL) of the transmission line. The method monitors the sum of the dual-ended measured currents of the interested line, and trip if the sum exceeds a certain threshold or a certain percentage of the restraint current (“percentage restraint” current differential). However, there are large charging capacitive currents in the long distance transmission line and underground cables, which may result in the failure of the line current differential protection [13]. In this case, proper charging current compensation must be considered with terminal voltage measurements. One way is to compensate the half of the total charging current at each end using the local voltage measurements [12]. The method considers the line as a generalized lumped \( \pi \) model. However, the model is not quite accurate and it cannot fully compensate the capacitive current. In [14], a charging current compensation method with distributed parameter line model is proposed. However, this method decouples the three phase lines into sequence mode lines, with the assumption that the line is geometrically symmetrical, which may also result in inaccurate charging current compensation. Moreover, the line parameters are typically different for the different homogeneous sections in non-homogeneous transmission lines. In this case, the mode transformation matrices are usually different and the overall mode network of the non-homogeneous lines is extremely complex [15], which could bring more challenges for charging capacitive current compensation in non-homogeneous transmission lines.

In this paper, an improved current differential protection for non-homogeneous lines is proposed. Synchronized three phase voltage and current measurements at both terminals of the interest line are required. The three phase distributed line model with full consideration of line asymmetry is utilized to represent the relationship between the dual terminal voltage and current phasor measurements of one homogeneous section. Afterwards, the phase differential currents with complete capacitive current compensation are derived for the non-homogeneous transmission line. The effectiveness of the...
The proposed method is verified via a number of internal and external fault events. The simulation results show improved dependability and security of the proposed protection scheme compared to the legacy current differential method. The rest of the paper is organized as follows. Section II introduces the modeling method of the non-homogeneous transmission line and derives the principle of the proposed protection method. Section III demonstrates the simulation results. Section IV draws a conclusion.

II. PROPOSED PROTECTION METHOD

A. Modeling of Homogeneous Transmission Line Section

The main idea of the non-homogeneous transmission line modeling method is to combine the models of all homogeneous transmission line sections. For each homogeneous section, the line parameters are the same (homogeneous) in the section and are typically different from those of the other sections. In the previous work of the authors [15], a compact two terminal model of one homogeneous line section with fully distributed parameters is proposed, as shown in (1). Note that this model fully considers distributed parameters as well as asymmetry of transmission circuits.

\[ \begin{bmatrix} \vec{V}_{I_{0}} \\ \vec{I}_{I_{0}} \end{bmatrix} = \begin{bmatrix} I_{3\times 3} & 0 \\ 0 & Z \end{bmatrix} e^{j\omega t} \begin{bmatrix} I_{3\times 3} & 0 \\ 0 & Z \end{bmatrix} \begin{bmatrix} \vec{V}_{I_{0}} \\ \vec{I}_{I_{0}} \end{bmatrix} \]

where subscript left indicates left end of the line and subscript right indicates right end. The direction of two end currents is from the left end to the right end of the line. \( I_{3\times 3} \) is an identity matrix with the dimension of 3 x 3 (corresponding to the three phase system). \( l \) is the length of the section, \( R, L, G \) and \( C \) are series resistance, series inductance, shunt conductance and shunt capacitance matrices per unit length respectively.

\[ B = \left[ Z(G \cdot j\omega C) I_{3\times 3} \right], \quad e^\omega \text{ is defined as } \sum_{n=0}^{\infty} (B^\omega/n! \text{ and } Z = R + j\omega L.} \]

B. Principle of the Proposed Current Differential Protection

Figure 1. Two terminal non-homogeneous transmission line

An example two terminal non-homogeneous transmission line is shown in Figure 1. The example line contains two homogeneous sections (section 1 and section 2) and with an internal fault in section 1. In fact, the model in (1) describes the relationship between three phase voltages and currents at both ends. The two adjacent sections can be combined by observing that: (a) the two adjacent sections share the same phase voltages at the common nodes, and (b) the current flowing into the common nodes satisfy the KCL. With this methodology, the current differential protection scheme could be designed.

For the section 2, it is a homogeneous section and can be represented as (2):

\[ \begin{bmatrix} \vec{V}_{I_{0}} \\ \vec{I}_{I_{0}} \end{bmatrix} = \begin{bmatrix} I_{3\times 3} & 0 \\ 0 & Z \end{bmatrix} e^{j\omega t} \begin{bmatrix} I_{3\times 3} & 0 \\ 0 & Z \end{bmatrix} \begin{bmatrix} \vec{V}_{I_{0}} \\ \vec{I}_{I_{0}} \end{bmatrix} \]

where, \( Z_2, B_2 \) are parameter matrices of section 2 and \( l_2 \) is the entire length of section 2.

For the section 1, the left and right parts of the fault circuit are two homogeneous sections, which can be represented as (3) and (4):

\[ \begin{bmatrix} \vec{V}_{I_{0}} \\ \vec{I}_{I_{0}} \end{bmatrix} = \begin{bmatrix} I_{3\times 3} & 0 \\ 0 & Z \end{bmatrix} e^{j\omega t} \begin{bmatrix} I_{3\times 3} & 0 \\ 0 & Z \end{bmatrix} \begin{bmatrix} \vec{V}_{I_{0}} \\ \vec{I}_{I_{0}} \end{bmatrix} \]

\[ \begin{bmatrix} \vec{V}_{I_{0}} \\ \vec{I}_{I_{0}} \end{bmatrix} = \begin{bmatrix} I_{3\times 3} & 0 \\ 0 & Z \end{bmatrix} e^{j\omega t} \begin{bmatrix} I_{3\times 3} & 0 \\ 0 & Z \end{bmatrix} \begin{bmatrix} \vec{V}_{I_{0}} \\ \vec{I}_{I_{0}} \end{bmatrix} \]

where \( Z_1 \) and \( B_1 \) are parameter matrices of section 1, \( l_1 \) is the entire length of section 1 and \( I_i \) is the distance between the fault location and receiving end.

Define the three phase fault current \( \overrightarrow{I}_f \), whose direction is flowing out the node at the fault location. Therefore, the KCL at the location of the fault can be represented as (5),

\[ \overrightarrow{I}_f = \overrightarrow{I}_f + \overrightarrow{I}_f \]

Substitute (5) and (4) into (3),

\[ \begin{bmatrix} \vec{V}_{I_{0}} \\ \vec{I}_{I_{0}} \end{bmatrix} = \begin{bmatrix} I_{3\times 3} & 0 \\ 0 & Z \end{bmatrix} e^{j\omega t} \begin{bmatrix} I_{3\times 3} & 0 \\ 0 & Z \end{bmatrix} \begin{bmatrix} \vec{V}_{I_{0}} \\ \vec{I}_{I_{0}} \end{bmatrix} \]

Define the following matrices,

\[ \begin{bmatrix} I_{3\times 3} & 0 \\ 0 & Z \end{bmatrix} e^{j\omega t} \begin{bmatrix} I_{3\times 3} & 0 \\ 0 & Z \end{bmatrix} \]

where \( N_0, M_0 \) and \( P_0 \) \( (i, j=1, 2) \) are 3 x 3 submatrices.

Therefore, with (2), (6), (7), (8) and (9), \( I_{3\times 3} + I_{3\times 3} = M_{3\times 3} \cdot V_{R} - M_{3\times 3} \cdot I_{N} - M_{3\times 3} \cdot I_{P} + P_{3\times 3} \cdot I \)

From the KCL at the common node of section 1 and section 2,

\[ \overrightarrow{I}_{T_2} + \overrightarrow{I}_{T_3} = 0 \]

Comparing the (10) and (11),

\[ M_{3\times 3} \cdot V_{R} - M_{3\times 3} \cdot I_{N} - M_{3\times 3} \cdot I_{P} + P_{3\times 3} \cdot I = 0 \]

From (12), when the line is under healthy condition, \( I_f = 0 \), \( (M_{3\times 3} \cdot V_{R} - M_{3\times 3} \cdot I_{N} - M_{3\times 3} \cdot I_{P} + P_{3\times 3} \cdot I) = 0 \); when the line is under fault condition (e.g. with fault in section 1), \( (M_{3\times 3} \cdot V_{R} - M_{3\times 3} \cdot I_{N} - M_{3\times 3} \cdot I_{P} + P_{3\times 3} \cdot I) = -P_{3\times 3} \cdot I_f = 0 \). Similar conclusion can be drawn when the fault is in section 2. Thus, the differential current \( I_{diff} \) and restraint current \( I_{res} \) can be designed as,

\[ I_{diff} = (M_{3\times 3} \cdot V_{R} - M_{3\times 3} \cdot I_{N} - M_{3\times 3} \cdot I_{P} + P_{3\times 3} \cdot I) \]

Note that the \( I_{diff} \) and \( I_{res} \) are two column vectors with three elements. The three values of \( I_{diff} \) and \( I_{res} \) correspond to phase A, B and C, respectively. In fact, the matrix \( P_{3\times 3} \) typically has a specific structure that the diagonal elements in \( P_{3\times 3} \) are much larger than the other values. Therefore, from (12), for the differential current \( I_{diff} \), the entries corresponding to the faulted phases are usually much larger than the entries corresponding to the other phases. This feature can be adopted for faulted phase selection.

C. Protection Algorithm

In section II.B, the three phase differential current \( I_{diff} \) and restraint current \( I_{res} \) with full compensation of the capacitive currents are obtained. In this section, the detailed protection algorithm is presented. The method follows the idea of the percentage restraint current differential protection,
with the dual slope characteristic [12] as depicted in Figure 2. The method monitors the three phases “in parallel”, which means that the differential and restraint currents of each phase are monitored according to Figure 2 at the same time: an internal fault is detected if for any phase the following condition is satisfied,

\[ I_{\text{diff}} > I_{\text{set1}}, I_{\text{set1}} \leq I_{\text{set2}} \] OR \[ I_{\text{diff}} > K_1 I_{\text{set1}}, I_{\text{set1}} > I_{\text{set2}} \] \( (15) \)

where \( I_{\text{diff}} \) and \( I_{\text{res}} \) are the differential and the restraint currents of one phase, \( I_{\text{set1}} \), \( I_{\text{set2}} \), \( K_1 \), \( K_2 \) are user defined values (settings of the relay) and here \( K_1 = 0 \) is adopted in \( (15) \). The current differential protection issues the trip signal when the internal fault is detected.

![Figure 2. Dual slope restraint characteristic](image)

### III. SIMULATION

The example non-homogeneous transmission line, consisting of an overhead line and an underground cable as shown in Figure 3, is utilized to validate the effectiveness of the proposed current differential protection method. The system is with the 500kV rated voltage, the 1000MVA rated capacity, and the 50Hz nominal frequency. Section 1 of the line is an underground cable with the length of 120km, while section 2 of the line is an overhead line with the length of 180km. Both the overhead line and underground cable are untransposed, and the circuit is built in PSCAD/EMTDC using the frequency-dependent (phase) model. The single phase cable structure of the section 1 and the tower structure of the section 2 are also shown in Figure 3. Three phase voltage and current synchrophasor measurements are installed at terminals of the line. These phasors are calculated according to the IEEE C37.118 standard, using sample value measurements from PSCAD/EMTDC. To make the experiment more practical, Gaussian distributed measurement errors with 1% p.u. standard deviation and zero mean are added to the current and voltage instantaneous measurements. To verify the performance of the proposed current differential protection method, a large amount of typical fault cases are simulated in the example system. Due to the space limitations, the paper only demonstrates the protection results of the 4 fault events: internal faults \( F_1 \sim F_3 \) and external fault \( F_4 \). For each event, the performance of the legacy current differential method is also presented as a comparison to the proposed method. Here the common charging current compensation method [16-17] is taken as an example for the legacy current differential scheme. The method utilizes a lumped \( \pi \) model, where the charging capacitive currents at each terminal can be approximated as half of total shunt admittance matrix of the line multiplied by the three-phase voltage vector at that terminal. The differential current \( I_{\text{diff}} \) and restraint current \( I_{\text{res}} \) of the legacy method can be calculated as,

\[ I_{\text{diff}} = |I_1 - I_R - 0.5Y_{\text{total}} V_1 + 0.5Y_{\text{total}} V_R| \]
\[ I_{\text{res}} = |I_1 + I_R - 0.5Y_{\text{total}} V_1 - 0.5Y_{\text{total}} V_R| \]

where \( Y_{\text{total}} = (G_1 + jwC_1)I_1 + (G_2 + jwC_2)I_2 \), \( G_1 \), \( C_1 \) and \( G_2 \), \( C_2 \) are shunt conductance and capacitance matrices per unit length of the section 1 and section 2 respectively. The legacy method models the non-homogeneous transmission line as a lumped \( \pi \) model, while the proposed method utilizes fully distributed model in matrix form to fully compensate the charging current without any assumption about the line structures. In order to make the two methods comparable, the settings for both methods (according to typical settings of current differential relays) are the same, as follows, \( K_1 \) is set as 0 and \( K_2 \) is set as 0.2, \( I_{\text{set1}}, I_{\text{set2}} \) are set as 0.2kA, 1kA respectively. Also, for each event, \( \text{trip} = 1 \) means the trip signal is issued.

![Figure 3. Example transmission system](image)

#### A. Internal Fault \( F_1 \): Low Impedance A-G Fault in Section 2

Phase A to ground internal fault \( F_1 \) occurs at 80km away from the terminal R with the 0.01 ohm fault impedance and at time 0.3s. The three phase differential currents \( I_{\text{diff}} \) of the proposed method and the legacy method are depicted in the Figure 4. The title of Figure 4 shows the meaning of \( x \) axis and \( y \) axis of each subfigure; three subfigures from top to bottom represent the phase ABC receptively (same conventions hold for Figure 5 to Figure 15). From Figure 4, it can be found that the phase differential currents of the proposed method are less than 10 A while the phase differential currents of the legacy method are larger than 100 A before the fault \( F_1 \) occurs (time less than 0.3 s). This implies that the proposed method is with more accurate charging current compensation.

During the internal fault \( F_1 \), the phase A differential currents of the both methods are larger than 6.6 kA after one cycle, and the phase BC differential currents are within the normal range.

![Figure 4. Phase differential current of the proposed method and the legacy method, low impedance A-G fault](image)

Next, Figure 5 and Figure 6 depict the protection results of the proposed method and legacy method. Figure 5 depicts the traces of the restraint currents and the differential currents of the proposed method and legacy method. From Figure 5, it can be found that the phase A traces of both methods enters the trip region, while the phase B and phase C traces keep in the block region. Figure 6 depicts the trip signal of the both methods. The proposed method issues trip signal at 0.30325s and legacy method issues at 0.3035s for phase A. For the phase B and phase C, the trip signals are not issued. The performances of the proposed method and the existing method are comparable during this low impedance A-G fault (the proposed method operates 0.25ms faster than the legacy method).
B. Internal Fault F\textsubscript{2}: Low Impedance AB-G Fault in Section 1

Phase A and B to ground internal fault F\textsubscript{2} occurs at 80 km away from the terminal S with the 0.01 ohm fault impedance and at time 0.3s. The three phase differential currents of the proposed method and the legacy method are depicted in the Figure 7. Figure 8 and Figure 9 depict the protection results of the proposed method and legacy method.

From Figure 7, it can be found that differential currents of the both methods for phase A and B are near 10 kA one cycle after the fault F\textsubscript{2} occurs. From Figure 8, phase A and phase B trip signals enter the protection region, and phase C traces keep in the block region. One can observe that the blue trace is closer to the y axis compared to the red trace, i.e. the blue trace corresponds to lower restraint current and higher differential current, and therefore improved dependability during internal faults. From Figure 12, for the proposed method, phase A trip signals of the proposed and legacy method are issued at 0.31225s and 0.31425s, respectively. The phase B and C trip signals are not issued for both methods. The proposed method trips this internal fault 2.25ms faster than the legacy method.

C. Internal Fault F\textsubscript{3}: High Impedance A-G Fault in Section 2

Phase A to ground internal fault F\textsubscript{3} occurs at 80 km away from the terminal R with the 300 ohm fault impedance and at time 0.3s. The three phase differential currents of the proposed method and the legacy method are depicted in the Figure 10. Figure 11 and Figure 12 depict the protection results of the proposed method and legacy method.

From Figure 10, it can be found that phase A differential current of the proposed method increases to 0.92 kA while phase A differential current of the legacy method increases to 0.84 kA one cycle after the fault F\textsubscript{3} occurs. From Figure 11, phase A traces of the both method enter the protection region, and phase B and C traces keep in the block region. One can observe that the blue trace is closer to the y axis compared to the red trace, i.e. the blue trace corresponds to lower restraint current and higher differential current, and therefore improved dependability during internal faults. From Figure 12, for the proposed method, phase A trip signals of the proposed and legacy method are issued at 0.31225s and 0.31425s, respectively. The phase B and C trip signals are not issued for both methods. The proposed method trips this internal fault 2.25ms faster than the legacy method.

D. External Fault F\textsubscript{4}: Low Impedance 3-Ph Fault at Bus R

Three phase external fault F\textsubscript{4} occurs at bus R with 0.01 ohm fault impedance and at time 0.3s. The three phase differential currents of the proposed method and the legacy method are depicted in the Figure 13. Figure 14 and Figure 15 depict the protection results of the proposed method and legacy method.

From Figure 13, it can be found that three phase differential currents are less than 0.3 kA one cycle after the fault F\textsubscript{4} occurs for the proposed method and three phase differential currents are less than 0.7 kA one cycle after the fault F\textsubscript{4} occurs for the proposed method. From Figure 14, three phase traces of the proposed method keep in the block region. One can observe that the blue curve is further away from the boundary between the block and the trip region.
These facts imply that the proposed method has improved security towards external faults. From Figure 15, the trip signals are not issued for the both methods.

![Figure 11. Phase differential current and phase restraint current of the proposed method and the legacy method, high impedance A-G fault](image)

![Figure 12. Phase trip signal of the proposed method and the legacy method, high impedance A-G fault](image)

![Figure 13. Phase differential current of the proposed method and the legacy method](image)

![Figure 14. Phase differential current and phase restraint current of the proposed method and the legacy method](image)

IV. CONCLUSION

This paper proposes an improved current differential protection for non-homogeneous transmission lines. The capacitive charging current is fully compensated using the distributed parameter line model, with consideration of line asymmetry structure. The related constant matrices can be calculated offline with the line parameters of each section. The differential and restraint currents can be calculated via matrix multiplications and additions, without adding much calculation complexity compared to the legacy methods. Afterwards, the protection algorithm can be implemented with the percentage differential principle. Numerical experiments on a two-terminal non-homogeneous transmission line (overhead lines mixed with underground cables) demonstrate that the proposed method more accurately compensate the capacitive currents compared to the existing method. Additionally, compared to the existing method, the proposed method presents improved dependability towards internal faults and improved security towards external faults.

![Figure 15. Phase trip signal of the proposed method and the legacy method](image)

V. REFERENCE