Dynamic state estimation-based fault locating on transmission lines

Yu Liu, A.P. Sakis Meliopoulos, Zhenyu Tan, Liangyi Sun, Rui Fan

Abstract: Accurate fault locating minimises labour and outage time. Efforts to increase the accuracy of fault locating methods are on-going. In this study, a dynamic state estimation-based fault locating (EBFL) method is proposed. Best implementation requires GPS synchronised sample value measurements at both ends of the line. The dynamic state estimator operates on the sampled values using a detailed dynamic model of the line, which includes the fault location as a state. Specifically, the dynamic line model is represented as a multi-section transmission line model, inclusive of phase conductor type and size, shield wire(s) type and size, and tower ground impedances, integrated with the fault model. This study presents extensive numerical experiments demonstrating that the method has higher accuracy than traditional fault locating methods for different fault types, locations and impedances. Additionally, the method works for both two-terminal and three-terminal transmission lines.

1 Introduction

Accurate fault locating on transmission lines is valuable for operators and utility crews, since the amount of time spent searching for the fault can be minimised. Thus, the outage time, labour and costs can be reduced. Legacy fault locating techniques can be mainly classified into two groups: fundamental frequency phasor-based methods and travelling wave-based methods. A brief review of the two classes of methods is provided for two-terminal lines with commentary on the challenges when applied to three-terminal lines.

1.1 Fundamental frequency phasor-based methods

These methods use voltage and current phasors of fundamental frequency to calculate the impedance and afterwards the distance between the fault location and the terminals of the line [1–3]. They can be further classified into single-ended and dual-ended algorithms. (i) Single-ended algorithms calculate the distance to the fault from voltage and current phasors at one end. The main advantage is that it does not require communication channels from the remote end. The accuracy of these methods largely depends on the fault type and fault impedance, as well as the line model accuracy, presence of mutually coupled circuits, and grounding parameters of the line [4]. (ii) Dual-ended algorithms use voltage and current phasors at both ends of the line. They can be further subdivided into methods that use GPS synchronised measurements or non-synchronised measurements. These methods are, in general, more accurate than single-ended methods as the influence of fault impedance and mutually coupled circuits is mitigated [5, 6]. The algorithms are also characterised on the basis of the model they use: sequence model or phase-based asymmetric model. Another characteristic is the data used (three-phase voltages only, three-phase voltages and currents) [7–9]. Disadvantages of fundamental frequency phasor-based single-ended or dual-ended methods are as follows. First, since the fundamental frequency phasors filter out transients, the phasors may contain inaccuracies which propagate to the fault locating results if the system is experiencing transients. Second, most of these methods use a sequence line model (positive, negative and zero sequence) which is an approximate model resulting in increased fault locating error. Third, the grounding of the line is typically neglected (i.e. considered effectively grounded) and this can potentially generate large errors depending on the fault type.

1.2 Travelling wave-based methods

When a fault initiates, high-frequency travelling waves are generated and propagate away in both directions with approximately the speed of light. At each discontinuity, for example a bus with multiple lines, the fault etc. they reflect and transmit generating more travelling waves. Travelling wave-based methods [10, 11] monitor the travelling waves at one or both ends of the line and estimate the travelling time of these waves to determine the fault location. They can be classified into two groups: single-ended algorithms and dual-ended algorithms. (i) Single-ended algorithms use subsequent (reflected) travelling waves at one terminal to identify the fault location [12, 13]. They are classified as types A, C, E and F fault locators according to how the travelling waves can be generated. (ii) Dual-ended algorithms use the time differences between the arrival of the first wavefront at both terminals to determine fault location [13–15]. They are classified as types B and D fault locators according to how the two terminals can be synchronised with each other. Since the travelling waves are modulated by the frequency-dependent characteristics of the line, digital signal processing is required to identify the actual arrival time of the wavefront. Several methods have been applied, including wavelet transformation [15]. Travelling wave-based methods have the following disadvantages. First, the intensity of travelling waves is greatly influenced by the fault initiation time relative to the power frequency cycle. This generates the issue of detection reliability of the travelling waves. Second, they require special instrumentation since the usual current transformers (CTs) will filter out the high-frequency content of the travelling waves. Third, assuming the instrumentation does not alter the signal, very high sampling rates are needed in order to detect the wavefront with sufficient accuracy [16]. For example, a system with a 100 kilosamples/s sampling rate could cause up to 0.93 miles systematic error with type D fault locator.

1.3 Additional challenges for three-terminal lines

Further, researchers have been studying fault locating methods for three-terminal lines. These methods can also be categorised into fundamental frequency phasor-based methods and travelling wave-based methods.

i. Fundamental frequency phasor-based methods: One method transforms the three-terminal line into a two-terminal equivalent line, and then directly applies standard two-terminal...
The EBFL method requires a dynamic line model, a model of the method. While the method is equally applicable to cable with improved accuracy in locating faults on transmission lines.

The controls \( u \) and the parameters \( p \) of the component. Subsequently, we convert these equations into a set of algebraic and first-order differential equations with non-linearities not \( > 2 \). In general, this is achieved by introducing additional variables. We refer to this as the quadratisation procedure. If the model is linear or quadratic, the quadratisation procedure is not needed. In the case of a faulted line, the model is quadratic and therefore the quadratisation procedure is not needed. The equations are cast into a specific syntax, which has been named device state, control and parameter quadratised dynamic model (SCPQDM). The syntax for this model is as follows: (see (1)) where \( x(t), u(t) \) and \( p(t) \) represent the state, control and parameter vectors of the components, respectively; terminal currents are represented by \( i(t) \), and terminal voltages are included in the state \( x(t) \). The second and third equations correspond to internal constraints that describe the relationship among states, controls and parameters of the components.

For the proposed fault locating application, the detailed SCPQDM of a faulted transmission line, including two- and three-terminal lines, is provided in Sections 9.1 and 9.2 of the Appendix, respectively. Note that a multi-section time-domain transmission line model is adopted. This model is a close approximation of the fully distributed transmission line model with explicit grounding representation [27], as shown in Section 9.3 of the Appendix. The multi-section model is computational preferable for fault locating purposes.

The device SCPQDM is integrated using the quadratic integration method [28] converting it into an algebraic companion form. The integration converts the SCPQDM into a set of algebraic equations of degree not higher than two in terms of the state, control and parameter variables. We refer to this model as the device state, control and parameter algebraic quadratic companion form (SCPAQCF) model.

The device SCPAQCF model has the format shown in (2) where all the matrices can be expressed by the matrices defined in device SCPQDM, \( \Delta t \) is the sampling interval and \( \Delta x_n = t - \Delta t \). It is noted that in our application, the transmission line does not have controls and therefore control variables are not present.

Any measurement on the line can be expressed as a function of the state, control and parameter vectors using the SCPAQCF model. For example, the model of phase A current measurement at the first terminal of the line at time \( t \) will be the first equation of the SCPAQCF model in (2). We refer to this as the measurement SCPAQCF object. The measurement SCPAQCF object has the format as shown in (3): (see (2) and (3)) (see (3))

The measurements \( i(t_n) \) are categorised into the following three types: actual, pseudo and virtual measurements. (i) Actual measurements are those obtained by actual metres such as terminal three-phase voltages and currents. They include a measurement error depending on the metre used which is quantified with a standard deviation. (ii) Pseudo measurements are introduced to ensure observability of the system, such as terminal neutral voltages. These measurements are often chosen as a best guess (e.g. zeros for neutral voltages) with large standard deviation. (iii) Virtual measurements correspond to internal constraints of the

\[
i(t) = Y_{eqx} x(t) + Y_{equ} u(t) + Y_{eqp} p(t) + D_{eqxn} \frac{dx(t)}{dt} + C_{eqx}
\]

\[
0 = Y_{eqx} x(t) + Y_{equ} u(t) + Y_{eqp} p(t) + D_{eqxn} \frac{dx(t)}{dt} + C_{eqx}
\]

\[
0 = Y_{eqx} x(t) + Y_{equ} u(t) + Y_{eqp} p(t) + \begin{bmatrix} x(t)^T (F'_{eqx}) x(t) \\ \vdots \\ \vdots \\ p(t)^T (F'_{eqq}) p(t) + u(t)^T (F'_{eqx}) x(t) \end{bmatrix}
\]

\[
0 = Y_{eqx} x(t) + Y_{equ} u(t) + Y_{eqp} p(t) + C_{eqx}
\]
The DSE algorithm provides the best estimate of the extended state from the measurements \( z(t, m) \) described in (3). Assuming that the fault location is invariant during the fault, the following additional constraints are introduced to the DSE:

\[
0 = p(i) - p(t_m) \quad (4)
\]

The detail DSE algorithm can be found in [25, 26]. As an example, the unconstrained weighted least-square algorithm is

\[
x\hat{p}(t, m)^{t+1} = x\hat{p}(t, m)^t - (H^T W H)^{-1} H^T W (h(x\hat{p}(t, m)^t) - z) \quad (5)
\]

where the Jacobian matrix \( H = \partial h(x\hat{p}(t, m)) / \partial x \hat{p}(t, m) \).

The quality of the estimated \( x\hat{p}(t, m) \) is validated by the confidence level \( P_{\text{conf}}(t) \) from the chi-square test [29]. A high confidence level (near 100%) means the estimated results are trustworthy, i.e. the measurements fit the model well.

The DSE with fault location

In this section, we present the DSE algorithm where the distance to the fault is introduced as a state variable to be estimated. We refer to the resulting state as the extended state, \( x\hat{p}(t, m) = [x(t, m), p(t, m)]^T \), where \( p(t, m) \) is the fault location at times \( t \) and \( t_m \). The DSE algorithm provides the best estimate of the extended state from the measurements \( z(t, m) \) described in (3).

\[
\begin{align*}
0 &= p(t) - p(t_m) \\
0 &= p(t) - p(t - 2\Delta t)
\end{align*}
\]

The detailed DSE algorithm can be found in [25, 26]. As an example, the unconstrained weighted least-square algorithm is

\[
x\hat{p}(t, m)^{t+1} = x\hat{p}(t, m)^t - (H^T W H)^{-1} H^T W (h(x\hat{p}(t, m)^t) - z) \quad (5)
\]

where the Jacobian matrix \( H = \partial h(x\hat{p}(t, m)) / \partial x \hat{p}(t, m) \).

The quality of the estimated \( x\hat{p}(t, m) \) is validated by the confidence level \( P_{\text{conf}}(t) \) from the chi-square test [29]. A high confidence level (near 100%) means the estimated results are trustworthy, i.e. the measurements fit the model well. The confidence level \( P_{\text{conf}}(t) \) can be calculated by

\[
P_{\text{conf}}(t) = P(\chi^2 \geq \zeta(t)) = 1 - P(\zeta(t), m) \quad (6)
\]

\[
\zeta(t) = (h(x\hat{p}(t, m)) - z(t, m))^T W (h(x\hat{p}(t, m)) - z(t, m)) \quad (7)
\]

where \( P(\zeta(t), m) \) is the probability of \( \chi^2 \) distribution given \( \chi^2 \leq \zeta(t) \) with degree of freedom \( m \), \( W = \text{diag}(1/\sigma_1^2, 1/\sigma_2^2, \ldots) \), where \( \sigma_i \) is the measurement error standard deviation, and \( h(x\hat{p}(t, m)) \) is defined as in (3).

The overall flowchart of the DSE is provided in Fig. 1.

Fig. 1 Flowchart of DSE with fault location

system such as the second, third, fifth and sixth sets of equations in (2). Virtual measurements are treated as constraints in the DSE algorithm or the constraints are relaxed and treated as measurements with very small standard deviation.
4 Simulation results

We compare the performance of the proposed EBFL method with legacy fault locating schemes using two example test systems: (i) two-terminal transmission line (Section 4.1) and (ii) three-terminal transmission line (Section 4.2). The legacy fault locating schemes are referring to the IEE standard in [12]. A number of fault events have been simulated and the results have been stored in COMTRADE files for experimentation and performance evaluation of the proposed method. The data have been stored with a sampling rate of 80 samples/cycle or 4.8 kilosamples/s (sampling interval of 208.33 ms). Note that the IEC 61850–9-2LE defines two sampling rates, i.e. 80 and 256 samples/cycle.

4.1 Example test system 1: two-terminal transmission line

The first example test system is partially illustrated in Fig. 2. The full network used for the simulations is not shown. The line of interest (A1–A2) is a 500 kV, 135.22 mile-long transmission line. Three-phase voltage and current measurements are installed at both terminals of the transmission line.

We assumed that legacy fundamental frequency phasor-based single-ended and dual-ended fault locating schemes are applied to this line to estimate the fault distance \( m \) from side A1. Note we do not consider travelling wave-based methods because the sampling rate is too low for estimating the fault location with reasonable accuracy. The two legacy methods for comparison are as follows: (i) single-ended method, \( m = \text{Im}(V_{A1}/I_{A1})/\text{Im}(Z_{A1}) \); (ii) dual-ended method, \( m = (V_{A1} - V_{A2} + Z_{L1}I_{A1})/(Z_{L1}(I_{A1} + I_{A2})) \); where \( Z_{L1} \) is the positive (negative) sequence impedance of the line; the values of \( V_{A1} \) and \( I_{A1} \) are chosen according to different types of faults, as shown in Table 1. In Table 1, for side \( A1 \) (\( i = 1, 2 \) for sides A1 and A2, respectively) are chosen voltage and current phasors of each phase, \( I_{A1,i} = (I_{A1,i} + I_{A2,i} + I_{A2,i})/3 \) is the zero-sequence current; \( k_0 = (Z_{d0} - Z_{L2})/Z_{d0} \) is the zero-sequence compensation factor; and \( Z_{d0} \) is the zero-sequence impedance of the line.

We compare the performance of the above legacy fault locating schemes to the EBFL method with different test cases as described below. Due to space limitations, here we will only present results for phase A to neutral faults and phases A–B faults. Results for other fault types are similar.

4.1.1 Test case 1: bolted phase A–N faults: A 0.01 \( \Omega \) bolted phase A to neutral fault occurred on lines A1–A2, 50 miles from bus A1. Figs. 3a and b depict the fault locating results of legacy single-ended and dual-ended fundamental frequency phasor-based methods. The fault locating results are 51.2722 miles from bus A1 for the legacy single-ended method and 49.6947 miles from bus A1 for the legacy dual-ended method.

Fig. 4a shows the fault locating results of the proposed EBFL method. The fault is computed to be at 50.0461 miles from bus A1. Note that the average error is 0.0461 miles, which is much smaller compared to legacy methods (1.2722 miles for legacy single-ended method and 0.3053 miles for legacy dual-ended method). Fig. 4b demonstrates the \( \chi^2 \) test results of the DSE process. A 100% confidence level proves that all the states and parameters of the system are consistent with the model, and the fault locating results are trustworthy. Fig. 4c provides the standard deviation of the calculated fault location (0.0878 miles), which proves the stability of the method. From these figures, we can see that for this fault the proposed EBFL method is more accurate than both legacy methods.
To further validate the effectiveness of the proposed EBFL method, 0.01 Ω bolted phase A to neutral faults at different fault locations are tested. The results are provided in Fig. 5a. It is observed that the EBFL method is more accurate than both legacy methods.

4.1.2 Test case 2: Impedance phase A–N faults: 10 ohms impedance phase A to neutral faults at different fault locations are tested. The results are provided in Fig. 5b. It is observed that EBFL method is more accurate than both legacy methods.

4.1.3 Test case 3: Bolted phases A–B faults: 0.01 Ω bolted phases A–B faults at different fault locations are tested. The results are provided in Fig. 5c. It is observed that EBFL method is more accurate than both legacy methods.

4.1.4 Test case 4: Impedance phases A–B faults: 10 ohms impedance phases A–B faults at different fault locations are tested. The results are provided in Fig. 5d. It is observed that the EBFL method is more accurate than both legacy methods.

4.2 Example test system 2: Three-terminal transmission line

The second example test system is partially shown in Fig. 6. The line of interest is a 500 kV three-terminal transmission line. The lengths of branch 1 (A1–T), branch 2 (A2–T) and branch 3 (A3–T) are shown in the figure. Here three-phase voltage and current measurements are installed at all three terminals of the transmission line (A1, A2 and A3). The rest of the network is not shown.

We assumed that legacy fundamental frequency phasor-based fault locating schemes are applied to this line to estimate the location of the fault. The legacy method first estimates the branch that the fault locates in, and next finds the location of the fault inside that specific branch. It simply estimates the voltage phasor at tap T from the terminal voltage and current phasors of two healthy branches; the branch with fault is the branch which has different estimation of the voltage phasors at tap T. Afterwards, the three-terminal line fault locating problem is equivalently transferred into a two-terminal line fault locating problem. Similarly as in Section 4.1, single-ended and dual-ended methods can be used to determine the location of the fault.

The reason we do not consider travelling wave-based legacy methods is that (i) the sampling rate here is too low for an estimated fault location with reasonable accuracy, and (ii) travelling wave-based methods have limitations in three-terminal lines since they cannot differentiate the reflection from the tap point and from the terminal of lines.

For our proposed EBFL on three-terminal lines, one important step is to determine the branch that the fault locates in. This is achieved by building three different models where each model corresponds to the fault inside each branch. Three models are running simultaneously with all available measurements and two of the fault locating results will generally converge to unrealistic values (less than zero or larger than the whole length of the branch) or simply diverge.

We compare the performance of the legacy fault locating schemes to EBFL method with different test cases as described below. Due to space limitations, here we will only present results for phase A to neutral faults and phases A–B faults, inside branch 1. Nevertheless, the results are similar for other fault types and fault locations.

4.2.1 Test case 5: Bolted phase A–N faults in branch 1: 0.01 Ω bolted phase A to neutral faults at different fault locations in branch 1.
branch 1 are tested. The results are provided in Fig. 7a. It is observed that EBFL method is more accurate than both legacy methods.

4.2.2 Test case 6: impedance phase A–N faults in branch 1:
10 ohms impedance phase A to neutral faults at different fault locations in branch 1 are tested. The results are provided in Fig. 7b. It is observed that the EBFL method is more accurate than both legacy methods.

4.2.3 Test case 7: bolted phases A–B faults in branch 1:
0.01 Ω bolted phases A–B faults at different fault locations in branch 1 are tested. The results are provided in Fig. 7c. It is observed that the EBFL method is more accurate than both legacy methods.

4.2.4 Test case 8: impedance phases A–B faults in branch 1:
10 ohms impedance phases A–B faults at different fault locations in branch 1 are tested. The results are provided in Fig. 7d. It is observed that the EBFL method is more accurate than both legacy methods.

5 Discussions

To further validate the proposed method, additional demonstrative results are provided in this section, including the performance of the method with different sampling rates and high fault impedances.

5.1 Performance with different sampling rates

In this section, the performance of the proposed EBFL algorithm with different sampling rates is demonstrated via the following example case: a group of 0.01 Ω phase A to neutral fault through the line in the system is shown in Fig. 2. The tested sampling rates include 300, 600, 1200, 2400, 3600, 4800 and 6000 samples/cycle. The results of the fault locating absolute error versus the actual fault location are shown in Fig. 8. Also, the fault locating results of the fundamental frequency phasor-based dual-ended legacy method are provided for comparison. Here the single-ended legacy method results are not included in the figure since they have much larger errors than the dual-ended method. It can be observed that (i) the absolute error of the proposed method is smaller with higher sampling rate; (ii) the EBFL method is more accurate than the dual-ended legacy method with sampling rate higher than 600 samples/s. It is recommended that the sampling rate of 4800 samples/s is used as it is consistent with the standard IEC 61850-9-2LE and in this case the error of the proposed method is sufficiently small.

5.2 Performance with high fault impedances

In this section, the performance of the proposed EBFL algorithm with high fault impedances is demonstrated via the following example case: a group of phase A to neutral fault through the line in the system is shown in Fig. 2, with sampling rate 4800 samples/s. The tested fault impedances include 0.01, 10, 40 and 100 Ω. The results of the fault locating absolute error versus the actual fault location and the fault impedance are shown in Fig. 9.
Fig. 9 Fault locating results comparison, different fault impedances, variable fault location

Similarly, here only the dual-ended legacy method is introduced for comparison since the single-ended legacy method has much larger errors. From the figure, it can be concluded that (i) the proposed EBFL method has more accurate fault locating results than dual-ended legacy method, for all fault impedances; (ii) unlike the dual-ended legacy method, the fault locating errors do not increase with higher fault impedances.

6 Conclusions

A DSE EBFL algorithm has been presented to accurately determine the fault location in transmission lines, for both two-terminal and three-terminal lines. First, the dynamic model of the faulted line (multi-section transmission line with fault) is built in an object-oriented SCPQDM/SCPAQCF syntax. A dynamic state estimator is used to compute the best estimation of the extended states of the faulted line. The extended states consist of the states of the line and the fault location. The proposed algorithm is validated via numerical simulations with different fault types, positions and impedances on two example test systems. Additional case studies are also provided with different sampling rates and high fault impedances.

The main advantages of the algorithm include: (i) it uses a high-fidelity multi-section transmission line dynamic model; (ii) it uses instantaneous values (sampled values) instead of phasors enabling estimation of the fault location in the presence of transients, which is very important when the fault duration is short; (iii) it demonstrates higher accuracy compared with legacy fault locating schemes; (iv) the sampling rate it requires is relatively low (in this case 4.8 kilosamples/s) and therefore it can be used with existing instrumentation in substations.

7 Acknowledgments

This work was supported by EPRI, PSERC and NYPA. Their support was greatly appreciated.

8 References


9 Appendices

9.1 SCPQDM of two-terminal transmission line with fault

This appendix describes the SCPQDM of a two-terminal multi-section transmission line integrated with a fault. The overall model is given in Fig. 10a. Here multi-section transmission line models are adopted for both the left and the right side parts, with m and n sections, respectively, where each section represents a π-equivalent short line model. The number of sections is chosen in such a way that the travelling distance of the electromagnetic wave during one sampling interval is comparable to the length of each section. The overall model can be obtained by combining the models of all sections. The π-equivalent models for each section inside the left and the right parts are provided in Figs. 10b and c. The matrices of SCPQDM for section k, left part are (all other matrices are zeros or null): (see equation below) where $y_{1k}(t)$, $y_{2k}(t)$, $y_{3k}(t)$ and $y_{4k}(t)$ are new states introduced to quadratise the model, $I_L$ is the identity matrix with the dimension of 4.
The matrices of SCPQDM for section \( k \), right part are (all other matrices are zeros or null): (see equation below) where \( y_{vk}^{(R)}(t) \) and \( y_{vk}^{(B)}(t) \) are new states introduced to quadratise the model.

### 9.2 SCPQDM of three-terminal transmission line with fault

This appendix describes the SCPQDM of a three-terminal multi-section transmission line integrated with a fault. The overall model in Fig. 11 consists of three parts: a two-terminal transmission line model with fault (same as in Section 9.1 of the Appendix) and two multi-section transmission line models without faults, with section number \( m + n \), \( p \) and \( q \), respectively. Here the section numbers are selected according to the same criteria as in Section 9.1 of the Appendix. Detail model for section \( k \) of the multi-section transmission line without fault can be found in [25, 26]. The overall model can be obtained by combining the models of all sections.

### 9.3 Validation of multi-section transmission line model

This appendix provides a comparison between the multi-section transmission line model and the fully distributed, frequency-dependent line model [27]. Owing to space limitations, the comparison is limited to an energisation event. The test system consists of a source, a line and a load as shown in Fig. 12a. The line is modelled with the multi-section model in one case and with the distributed, frequency-dependent model in the other case. The results during line energisation for both cases are shown for the period 0 s to 2 ms. The currents at the output of the source and the voltages at the middle point of the transmission line, for both cases, are depicted in Figs. 12b and c. It can be observed that the currents have the maximum difference of <1.3% while the voltages have the maximum difference of <2%. It is concluded that even for this severe transient event, the accuracy of the model is very good as compared to the distributed, frequency-dependent line model.

**Fig. 10** Model of two-terminal faulted line (a) Overall model; (b) \( \pi \)-equivalent model for section \( k \), left part; (c) \( \pi \)-equivalent model for section \( k \), right part

The matrices of SCPQDM for section \( k \), right part are (all other matrices are zeros or null): (see equation below) where \( y_{ak}^{(R)}(t) \),
\[ x(t) = [v_k^{(R)}(t) \ v_k^{(R)}(t) \ \dot{y}_k^{(R)}(t) \ y_k^{(R)}(t) \ \dot{y}_k(t) \ y_k(t)]^T; \]

\[ Y_{eq1} = \begin{bmatrix} G_k \cdot l/n & 0 & I_k & -G_k/l & 0 & 0 \\ 0 & G_k \cdot l/n & -I_k & 0 & -G_k/l & 0 \end{bmatrix}; \]

\[ D_{eq1} = \begin{bmatrix} C_k \cdot l/n & 0 & 0 & -C_k/l & 0 & 0 \end{bmatrix}; \]

\[ Y_{eq2} = \begin{bmatrix} -I_k & I_k & R_k \cdot l/n & 0 & 0 & -R_k/l \end{bmatrix}; \]

\[ D_{eq2} = \begin{bmatrix} 0 & 0 & L_k \cdot l/n & 0 & 0 & -L_k/l \end{bmatrix}. \]

\[ p(t) = l_f(t), \quad F_{eq}^i = f_{eq}^i = -1, \text{ others are 0, for } i = 1 - 12; \]

---

**Fig. 11** Model of three-terminal line with fault

---

**Fig. 12** Comparison between the multi-section model and the fully distributed model (a) Example test system, (b) Currents at the output of the source, (c) Voltages at the middle point of the line